Improved General Lower Bound for Spatially-Correlated Rician MIMO Capacity

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***Abstract*—In this letter we derive a new closed-form lower bound on the ergodic capacity of single-sided correlated Rician MIMO channels with arbitrary-rank mean matrices. The new bound is significantly tighter than previously reported bounds, and matches almost exactly with empirically-generated (exact) capacity results for all signal to noise ratios. The new closed-form bound is also more computationally efficient than the previous bounds in the literature.**

***Index Terms*—Capacity, MIMO, correlated, Rician.**

I. Introduction

RECENTLY, considerable research has focussed on multiple-input multiple-output (MIMO) wireless communication systems. These systems were initially shown to provide significant capacity improvements over single-antenna systems in uncorrelated Rayleigh fading [1]. Capacity results for more practical spatially-correlated Rayleigh [2, 3] and uncorrelated Rician MIMO channels have now emerged. For uncorrelated Rician channels, bounds were presented in [4, 5] for channels with rank-1 mean matrices, and exact expressions were derived in [6], but the results were not in closed form.

Only few analytical MIMO capacity results are available for spatially-correlated Rician channels. In [7–10], upper and lower bounds were presented for single-sided correlated Rician channels, with rank-1 mean matrices. In [11], these results were extended to arbitrary-rank means and double-sided correlation.

In this letter we derive a new closed-form lower bound on the capacity of single-sided correlated Rician MIMO channels with arbitrary-rank mean matrices. The new bound is significantly tighter than the previously reported bound in [11] which used a different bounding technique, and matches almost exactly with empirically-generated (exact) capacity results for all signal to noise ratios (SNR). The new closed-form bound is also more computationally efficient than the previous correlated Rician capacity bounds in [7–9, 11].

II. Capacity of Correlated Rician MIMO Channels

Consider a flat-fading $N_t \times N_r$ MIMO link modelled by

$$\mathbf{r} = \mathbf{H}\mathbf{a} + \mathbf{n} \tag{1}$$

where $\mathbf{r} \in \mathcal{C}^{N_r \times 1}$ is the received vector, $\mathbf{a} \in \mathcal{C}^{N_t \times 1}$ is the transmitted vector satisfying the power constraint $E[\mathbf{a}^\dagger \mathbf{a}] \leq P$, and $\mathbf{n} \in \mathcal{C}^{N_r \times 1}$ is noise $\sim \mathcal{CN}_{N_r}\left(\mathbf{0}_{N_r}, \sigma_n^2 \mathbf{I}_{N_r}\right)$. Also, $\mathbf{H} \in \mathcal{C}^{N_r \times N_t}$ is the MIMO channel matrix.

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The channel gains are assumed to undergo spatially-correlated Rician fading. The columns of $\mathbf{H}$ are complex Gaussian random vectors, each having the same Hermitian covariance matrix given by the receive correlation matrix $\mathbf{R}$. We allow the mean vectors of each of the columns of $\mathbf{H}$ to be different. The rows of $\mathbf{H}$ are also modelled as complex Gaussian random vectors (transposed), each with covariance matrix given by the transmit correlation matrix $\mathbf{S}$. We assume $\mathbf{R}$ and $\mathbf{S}$ are positive-definite full-rank. Under these assumptions, the channel may be decomposed as

$$\mathbf{H} = \sqrt{a}\,\mathbf{M} + \sqrt{b}\,\mathbf{R}^{\frac{1}{2}}\mathbf{H}_w\mathbf{S}^{\frac{1}{2}} \sim \mathcal{CN}_{N_r,N_t}\left(\sqrt{a}\,\mathbf{M},\, b\mathbf{R}\otimes\mathbf{S}\right)$$

where $\mathbf{M}$ is the arbitrary-rank mean matrix satisfying $\mathrm{tr}\left(\mathbf{M}\mathbf{M}^\dagger\right) = N_r N_t$, a and b are power normalization coefficients, and $\mathbf{H}_w \sim \mathcal{CN}_{N_r,N_t}\left(\mathbf{0}_{N_r \times N_t}, \mathbf{I}_{N_r} \otimes \mathbf{I}_{N_t}\right)$.

In this paper we assume that spatial correlation between antenna elements only occurs at one end of the transmission link (i.e. single-sided correlation). In particular, we consider correlation occurring at the end with the least number of antennas. We also assume that the receiver has perfect channel knowledge and that the transmitter has no knowledge. As such, the transmitter operates under the assumption of i.i.d. Rayleigh channels (as discussed in [5], and used in [2, 3, 7] and elsewhere) and the total power P is evenly distributed across all transmit antennas. Under these assumptions, the ergodic MIMO capacity (in b/s/Hz) $C = E\left[\mathcal{I}\right]$, where $\mathcal{I}$ is the mutual information[1], is given by [1]

$$C = E\left[\log\left|\mathbf{I}_n + \frac{\gamma}{N_t}\bar{\mathbf{W}}\right|\right] = E\left[\log\left|\mathbf{I}_n + \frac{\gamma}{N_t}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^\dagger\right|\right] \tag{2}$$

where $\gamma = \frac{P}{\sigma_n^2}$ is the SNR, and

$$\tilde{\mathbf{H}} \triangleq \sqrt{a}\bar{\mathbf{M}} + \sqrt{b}\boldsymbol{\Lambda}^{\frac{1}{2}}\bar{\mathbf{H}}_w \sim \mathcal{CN}_{n,m}\left(\sqrt{a}\bar{\mathbf{M}}, b\boldsymbol{\Lambda}\otimes\mathbf{I}_m\right) \tag{3}$$

where $\boldsymbol{\Lambda}$ and $\bar{\mathbf{M}}$ are defined as

$$\boldsymbol{\Lambda} \triangleq \begin{cases} \mathbf{R} & \text{for } N_r \leq N_t \\ \mathbf{S} & \text{for } N_r > N_t \end{cases} \qquad \bar{\mathbf{M}} \triangleq \begin{cases} \mathbf{M} & \text{for } N_r \leq N_t \\ \mathbf{M}^\dagger & \text{for } N_r > N_t \end{cases}$$

Also, $n = \min(N_r, N_t)$, $m = \max(N_r, N_t)$, and $\bar{\mathbf{H}}_w \sim \mathcal{CN}_{n,m}\left(\mathbf{0}_{n\times m}, \mathbf{I}_n \otimes \mathbf{I}_m\right)$.

We now derive a tight lower bound on the capacity (2), which is the main result of the paper.

III. Improved Lower Bound

Theorem 1: The ergodic capacity in b/s/Hz of $N_t \times N_r$ Rician MIMO channels with rank-L mean matrix $\mathbf{M}$, and

[1]Note that if the transmitter knew the channel statistics, the true ergodic capacity would require a maximization over the input covariance matrix.

$$(\mathbf{A}_{\alpha_k,\ell})_{i,j} = \begin{cases} \tilde{\theta}_i^{j-1} & \text{for } i \neq \ell \\ \tilde{\theta}_i^{j-1}\left(\ln(\tilde{\theta}_i) - \mathrm{E_i}(-\tilde{\theta}_i) + \psi(\mathcal{V}_{\alpha_k,j}) + \sum_{t=1}^{\mathcal{V}_{\alpha_k,j}-1} \frac{(t-1)!}{(-\tilde{\theta}_i)^t}\left(e^{-\tilde{\theta}_i} - \binom{\mathcal{V}_{\alpha_k,j}-1}{t}\right)\right) & \text{for } i = \ell \end{cases} \quad (6)$$

correlation matrix $\mathbf{\Lambda}$ at the end with the least antennas is lower bounded by

$$C \geq \log\left(1 + \sum_{k=1}^{n}\left(\frac{\gamma}{N_t}\right)^k \exp\left(\sum_{t=0}^{k-1}\psi(m-t)\right) \times \sum_{\{\alpha_k\}}\left|b\mathbf{\Lambda}_{\alpha_k}^{\alpha_k}\right| \exp\left(\frac{\sum_{\ell=1}^{L_{\alpha_k}}|\mathbf{A}_{\alpha_k,\ell}|}{\prod_{i<j}(\tilde{\theta}_j - \tilde{\theta}_i)}\right)\right) \quad (4)$$

where $\psi(\cdot)$ is the digamma function, defined in [12, Def. 6.3.1], $\{\alpha_k\}$ is the set of all ordered $\alpha_k \subseteq \{1,\ldots,n\}$ with cardinality k, and $\tilde{\theta}_1,\ldots,\tilde{\theta}_{L_{\alpha_k}}$ are the non-zero eigenvalues of the rank-L_{α_k} matrix[2]

$$\tilde{\mathbf{\Theta}}(\alpha_k) = \frac{a}{b}\left(\mathbf{\Lambda}_{\alpha_k}^{\alpha_k}\right)^{-1}\bar{\mathbf{M}}_{\{1,\ldots,m\}}^{\alpha_k}\left(\bar{\mathbf{M}}_{\{1,\ldots,m\}}^{\alpha_k}\right)^{\dagger} \quad (5)$$

Also, $\mathbf{A}_{\alpha_k,\ell}$ is an $L_{\alpha_k} \times L_{\alpha_k}$ matrix with $(i,j)^{\text{th}}$ element $(\mathbf{A}_{\alpha_k,\ell})_{i,j}$ defined in (6), where $\mathcal{V}_{\alpha_k,j} \triangleq m - L_{\alpha_k} + j$, and $\mathrm{E_i}(\cdot)$ is the exponential integral, defined in [13, Def. 8.211].

Proof: We begin by applying the general bounding approach in [3] to (2), which yields

$$C \geq \log\left(1 + \sum_{k=1}^{n}\left(\frac{\gamma}{N_t}\right)^k \sum_{\{\alpha_k\}} \exp\left(E\left[\ln\left|\bar{\mathbf{W}}_{\alpha_k}^{\alpha_k}\right|\right]\right)\right) \quad (7)$$

We evaluate the expectations based on the moment generating function (m.g.f.) $\mathcal{M}_{\ln|\bar{\mathbf{W}}_{\alpha_k}^{\alpha_k}|}(s)$ as

$$\begin{aligned} E\left[\ln\left|\bar{\mathbf{W}}_{\alpha_k}^{\alpha_k}\right|\right] &= \frac{\mathrm{d}}{\mathrm{d}s}\mathcal{M}_{\ln|\bar{\mathbf{W}}_{\alpha_k}^{\alpha_k}|}(s)\Big|_{s=0} \\ &= \frac{\mathrm{d}}{\mathrm{d}s}\ln\left(E\left[\exp\left(s\ln\left|\bar{\mathbf{W}}_{\alpha_k}^{\alpha_k}\right|\right)\right]\right)\Big|_{s=0} \\ &= \frac{\mathrm{d}}{\mathrm{d}s}\ln\left(E\left[\left|\bar{\mathbf{W}}_{\alpha_k}^{\alpha_k}\right|^s\right]\right)\Big|_{s=0} \end{aligned} \quad (8)$$

where the second line followed from the definition of the m.g.f., and by recalling that $\mathcal{M}(0) = 1$. Note that in [10], the expectation in (8) was evaluated using Bartlett's Decomposition, which was limited to mean matrices of rank-1. From (3) and [11, Lem. 2] we note that $\bar{\mathbf{W}}_{\alpha_k}^{\alpha_k}$ has the complex non-central Wishart distribution

$$\bar{\mathbf{W}}_{\alpha_k}^{\alpha_k} \sim \mathcal{W}_k\left(m, b\mathbf{\Lambda}_{\alpha_k}^{\alpha_k}, \tilde{\mathbf{\Theta}}(\alpha_k)\right) \quad (9)$$

Using (9) and [11, Theor. 1] it can be shown that (8) becomes[3]

$$E\left[\ln\left|\bar{\mathbf{W}}_{\alpha_k}^{\alpha_k}\right|\right] = \ln\left|\mathbf{\Lambda}_{\alpha_k}^{\alpha_k}\right| + \sum_{t=0}^{k-1}\psi(m-t) + \frac{\frac{\mathrm{d}}{\mathrm{d}s}\left|{}_1F_1\left(\mathcal{V}_{\alpha_k,j}+s;\mathcal{V}_{\alpha_k,j};\tilde{\theta}_i\right)\tilde{\theta}_i^{j-1}\right|_{s=0}}{\mathrm{etr}\left(\tilde{\mathbf{\Theta}}(\alpha_k)\right)\prod_{i<j}(\tilde{\theta}_j - \tilde{\theta}_i)} \quad (10)$$

[2]We use the notation $\mathbf{A}_{\mathcal{G}}^{\mathcal{F}}$ to denote the submatrix of a $p \times q$ matrix $\mathbf{A}$, formed by taking only the rows indexed by $\mathcal{F} \subseteq \{1,\ldots,p\}$ and columns indexed by $\mathcal{G} \subseteq \{1,\ldots,q\}$.

[3]Here we use the compact notation for the determinant of an $L_{\alpha_k} \times L_{\alpha_k}$ matrix, written in terms of the i,j^{th} element.

where ${}_1F_1(\cdot)$ is the scalar confluent hypergeometric function

$${}_1F_1(a;b;x) \triangleq \sum_{k=0}^{\infty}\frac{\Gamma(a+k)}{\Gamma(a)}\frac{\Gamma(b)}{\Gamma(b+k)}\frac{x^k}{k!} \quad (11)$$

To evaluate the derivative in (10), we use a well-known formula for the derivative of a determinant, and note that ${}_1F_1(a;a;x) = {}_0F_0(x) = e^x$, to obtain

$$\frac{\mathrm{d}}{\mathrm{d}s}\left|{}_1F_1\left(\mathcal{V}_{\alpha_k,j}+s;\mathcal{V}_{\alpha_k,j};\tilde{\theta}_i\right)\tilde{\theta}_i^{j-1}\right|_{s=0} = \mathrm{etr}\left(\tilde{\mathbf{\Theta}}(\alpha_k)\right)\sum_{\ell=1}^{L_{\alpha_k}}|\mathbf{A}_{\alpha_k,\ell}| \quad (12)$$

where

$$(\mathbf{A}_{\alpha_k,\ell})_{i,j} = \begin{cases} \tilde{\theta}_i^{j-1} & \text{for } i \neq \ell \\ \tilde{\theta}_i^{j-1}e^{-\tilde{\theta}_i}\frac{\mathrm{d}}{\mathrm{d}s}{}_1F_1\left(\mathcal{V}_{\alpha_k,j}+s;\mathcal{V}_{\alpha_k,j};\tilde{\theta}_i\right)\Big|_{s=0} & \text{for } i = \ell \end{cases} \quad (13)$$

For the case $i = \ell$, we use (11) to obtain

$$(\mathbf{A}_{\alpha_k,\ell})_{i,j} = \tilde{\theta}_i^{j-1}e^{-\tilde{\theta}_i}\left(\sum_{t=0}^{\infty}\frac{\tilde{\theta}_i^t}{t!}\psi(\mathcal{V}_{\alpha_k,j}+t)\right) - \tilde{\theta}_i^{j-1}\psi(\mathcal{V}_{\alpha_k,j}) \quad (14)$$

To simplify the left-most term, which we now assign the name $\mathcal{L}$, we use [12, Eq. 6.3.2] to give

$$\begin{aligned} \mathcal{L} &= \tilde{\theta}_i^{j-1}e^{-\tilde{\theta}_i}\left(\sum_{t=0}^{\infty}\frac{\tilde{\theta}_i^t}{t!}\left(\sum_{r=1}^{\mathcal{V}_{\alpha_k,j}+t-1}\frac{1}{r}\right)\right) - \tilde{\theta}_i^{j-1}\mathcal{K} \\ &= \tilde{\theta}_i^{j-1}e^{-\tilde{\theta}_i}\left(\sum_{t=1}^{\infty}\frac{\tilde{\theta}_i^t}{t!}\left(\sum_{r=1}^{t}\frac{1}{r}\right)\right) \\ &\quad + \tilde{\theta}_i^{j-1}e^{-\tilde{\theta}_i}\left(\sum_{t=0}^{\infty}\frac{\tilde{\theta}_i^t}{t!}\left(\sum_{r=1}^{\mathcal{V}_{\alpha_k,j}-1}\frac{1}{r+t}\right)\right) - \tilde{\theta}_i^{j-1}\mathcal{K} \end{aligned} \quad (15)$$

where $\mathcal{K} \approx 0.5772$ is the Euler-Mascheroni constant. Next, using the definition of $\mathrm{E_i}(\cdot)$ [13], and combining [12, Eq. 6.5.4] and [12, Eq. 6.5.29], it can be shown that

$$\begin{aligned} \mathcal{L} &= \tilde{\theta}_i^{j-1}\left(\ln\left(\tilde{\theta}_i\right) - \mathrm{E_i}\left(-\tilde{\theta}_i\right) + e^{-\tilde{\theta}_i}\left(\sum_{t=0}^{\infty}\frac{\tilde{\theta}_i^t}{t!}\sum_{r=1}^{\mathcal{V}_{\alpha_k,j}-1}\frac{1}{r+t}\right)\right) \\ &= \tilde{\theta}_i^{j-1}\left(\ln\left(\tilde{\theta}_i\right) - \mathrm{E_i}\left(-\tilde{\theta}_i\right) + e^{-\tilde{\theta}_i}\left(\sum_{r=1}^{\mathcal{V}_{\alpha_k,j}-1}\left(-\tilde{\theta}_i\right)^{-t}g\left(t,-\tilde{\theta}_i\right)\right)\right) \end{aligned} \quad (16)$$

where $g(\cdot,\cdot)$ is the lower incomplete gamma function. Using [13, Eq. 8.352.1] we have

$$\begin{aligned}\mathcal{L} &= \tilde{\theta}_i^{j-1}\Bigg(\ln\left(\tilde{\theta}_i\right) - \mathrm{E_i}\left(-\tilde{\theta}_i\right) \\ &\quad + \sum_{t=1}^{\mathcal{V}_{\alpha_k,j}-1} \frac{(t-1)!}{(-\tilde{\theta}_i)^t}\left(e^{-\tilde{\theta}_i} - \sum_{r=0}^{t-1}\frac{(-\tilde{\theta}_i)^r}{r!}\right)\Bigg) \\ &= \tilde{\theta}_i^{j-1}\Bigg(\ln\left(\tilde{\theta}_i\right) - \mathrm{E_i}\left(-\tilde{\theta}_i\right) \\ &\quad + \sum_{t=1}^{\mathcal{V}_{\alpha_k,j}-1} \frac{(t-1)!}{(-\tilde{\theta}_i)^t}\left(e^{-\tilde{\theta}_i} - \binom{\mathcal{V}_{\alpha_k,j}-1}{t}\right)\Bigg) \qquad (17)\end{aligned}$$

The result follows by substituting (17) as the left-hand term in (14), and by combining (7), (10), and (12). ■

Corollary 1: At high SNR, (4) becomes

$$\begin{aligned}C \geq n \log\left(\frac{\gamma}{N_t}\right) + \frac{1}{\ln 2}\sum_{t=0}^{n-1}\psi\left(m-t\right) \\ + \log|b\mathbf{\Lambda}| + \frac{1}{\ln 2}\frac{\sum_{\ell=1}^{L}|\mathbf{A}_{\alpha_n,\ell}|}{\prod_{i<j}(\tilde{\theta}_j - \tilde{\theta}_i)} \qquad (18)\end{aligned}$$

Proof: This is obtained by noting that (4) is dominated by the n^{th} order term at high SNR. ■

It can be shown that (18) is consistent with the high-SNR lower bound we derived previously in [11] using a different bounding approach. The bounds in (4) and (18) are more efficient than the results in [11] (and rank-1 mean results in [7–9]), which are expressed in terms infinite series. We will also see in our numerical results that for low to medium SNRs the bound derived in this paper is significantly tighter than the bound in [11]. Moreover, for the special case $\mathbf{\Lambda} = \mathbf{I}_n$, it can also be shown that (18) is consistent with previous high-SNR results presented in [14].

IV. Numerical Results

We consider a common power normalization model with $a = K/(K+1)$ and $b = 1/(K+1)$, where K is the Rician K-factor which is the ratio of the power in the fixed (mean) component with respect to the average power in the fading components. Since $a + b = 1$, for fixed total transmit power the receiver SNR remains constant for any value of K. See [8, 9] for a comparison with another power normalization strategy which allows the receiver SNR to scale with K.

The mean and correlation matrices are generated using the practical channel model from [15].

Fig. 1 gives the lower bound (4) and empirically generated curves for 3×2, 6×4 and 9×6 systems. The (previous) lower bound from [11] is also presented for comparison. Receive correlation is assumed, with mean angle of arrival (AoA) $\theta_r = \frac{\pi}{2}$ and cluster angle spread $\sigma_r^2 = \frac{\pi}{16}$. The mean matrix is rank-2, and is constructed assuming two equal-power paths with $K = 10$, and with AoAs and angle of departures (AoD) offset from the mean AoA and AoD by $\pm\frac{\pi}{32}$. We see that the new lower bound (4) is significantly tighter than the bound from [11], and is almost exact for all antenna configurations, and over the entire range of SNRs. As expected from (18), both bounds converge to the empirical (exact) capacity at high SNR.

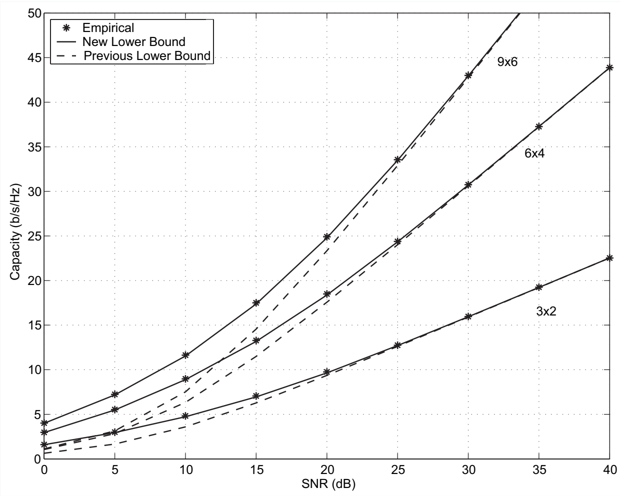

Fig. 1. Bounds and simulation results for ergodic capacity of correlated Rician MIMO channels for various antenna configurations, with Rician K-factor = 10, receiver correlation, rank-2 mean matrix. Receive correlation parameters are $\theta_r = \frac{\pi}{2}$, $\sigma_r^2 = \frac{\pi}{16}$ and $d_t = d_r = \frac{1}{2}$.

V. Conclusion

We have derived a new efficient closed-form lower bound on the capacity of single-sided correlated Rician MIMO channels with arbitrary-rank mean matrices. The new bound yields superior tightness over previously reported bounds, and matches almost exactly with the true capacity for all SNRs.

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