Asymptotic SER and Outage Probability of MIMO MRC in Correlated Fading

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***Abstract*—This letter derives the asymptotic symbol error rate (SER) and outage probability of multiple-input multiple-output (MIMO) maximum ratio-combining (MRC) systems. We consider Rayleigh fading channels with both transmit and receive spatial correlation. Our results are based on new asymptotic expressions that we derive for the p.d.f. and c.d.f. of the maximum eigenvalue of positive-definite quadratic forms in complex Gaussian matrices. We prove that spatial correlation does not affect the diversity order but that it reduces the array gain and hence increases the SER in the high SNR regime.**

***Index Terms*—Correlation, maximal ratio combining (MRC), multiple-input multiple-output (MIMO), Rayleigh channels.**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) maximal ratio-combining (MRC) systems have recently received much attention due to their ability to mitigate the severe effects of fading through diversity [1]. The performance of MIMO-MRC has been previously investigated in various uncorrelated and semi-correlated channel scenarios. The main performance measures evaluated have been symbol error rate (SER) and outage probability. Uncorrelated Rayleigh fading was considered in [2]–[7], and semi-correlated Rayleigh fading was considered in [8]–[10].

In this letter, we present results for double-correlated channels. In particular, we derive new asymptotic expressions for the SER and outage probability for arbitrary numbers of antennas, which are simple functions of the system and channel parameters. These expressions are particularly useful in comparison to the only other known double-correlated MIMO-MRC results, presented in [11], which in any case had SER expressions limited to two-antenna systems.[1] Moreover, the results we present here collapse trivially to semi-correlated scenarios and provide simpler expressions than the previous results in [8]–[10].

Our results hinge on new first-order expansions that we derive for the probability density function (p.d.f.) and cumulative distribution function (c.d.f.) of the maximum eigenvalue of positive-definite quadratic forms in complex Gaussian matrices. From these, we obtain a high SNR expression for the SER and determine the diversity order and array gain. In so doing, we prove that MIMO-MRC achieves the maximum spatial diversity order, and we quantify the increase in SER in the high SNR regime due to spatial correlation. We also analyze the outage probability at outage levels of practical interest and again quantify the effect of spatial correlation.

II. MIMO-MRC System Model

Consider an $N_r \times N_t$ MIMO system, where the $N_r \times 1$ received signal vector is

$$\mathbf{r} = \sqrt{\bar{\gamma}}\mathbf{H}\mathbf{w}x + \mathbf{n} \tag{1}$$

where x is the transmitted symbol with $E[|x|^2] = 1, \mathbf{w}$ is the beamforming (BF) vector with $E[||\mathbf{w}||^2] = 1, \mathbf{n} \in \mathcal{C}^{N_r \times 1}$ is additive noise vector $\sim \mathcal{CN}_{N_r,1}(\mathbf{0}_{N_r\times 1}, \mathbf{I}_{N_r})$, and $\bar{\gamma}$ is the transmit SNR. Also, $\mathbf{H}$ is the $N_r \times N_t$ channel matrix, assumed to be flat spatially correlated Rayleigh fading.

We assume that $\mathbf{H}$ can be decomposed according to the popular Kronecker correlation structure (as in [8], [10], and [12]) as follows:

$$\mathbf{H} = \mathbf{R}^{\frac{1}{2}}\mathbf{H}_w\mathbf{S}^{\frac{1}{2}} \tag{2}$$

where $\mathbf{R} > 0$ and $\mathbf{S} > 0$ are the receive and transmit spatial correlation matrices, respectively, with unit diagonal entries, and $\mathbf{H}_w \sim \mathcal{CN}_{N_r,N_t}(\mathbf{0}_{N_r\times N_t}, \mathbf{I}_{N_r} \otimes \mathbf{I}_{N_t})$.

The receiver employs the principle of MRC to give

$$z = \mathbf{w}^\dagger\mathbf{H}^\dagger\mathbf{r} = \sqrt{\bar{\gamma}}\mathbf{w}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{w}x + \mathbf{w}^\dagger\mathbf{H}^\dagger\mathbf{n} \tag{3}$$

and the SNR at the output of the combiner is derived as

$$\gamma = \bar{\gamma}\mathbf{w}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{w}. \tag{4}$$

The BF vector $\mathbf{w}$ is chosen to maximize this instantaneous output SNR, thereby minimizing the error probability. It is well known that the optimum BF vector $\mathbf{w}_{\text{opt}}$ is the eigenvector corresponding to the maximum eigenvalue $\lambda_{\max}$ of $\mathbf{H}^\dagger\mathbf{H}$. In this case, the output SNR (4) becomes

$$\gamma = \bar{\gamma}\mathbf{w}_{\text{opt}}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{w}_{\text{opt}} = \bar{\gamma}\lambda_{\max}. \tag{5}$$

Clearly, the performance of MIMO-MRC depends directly on the statistical properties of $\lambda_{\max}$. For double-correlated Rayleigh fading channels, $\lambda_{\max}$ is statistically equivalent to

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[1]Systems with two antennas at either the transmitter or the receiver.

the maximum eigenvalue of a positive-definite quadratic form in complex Gaussian random matrices $\mathbf{Y} \sim \mathcal{Q}_{n,m}(\mathbf{I}_n, \mathbf{\Omega}, \mathbf{\Sigma})$ (see [13] and [14] for more details), where $m = \max(N_r, N_t)$, $n = \min(N_r, N_t)$, and $\mathbf{\Omega} \in \mathcal{C}^{n\times n}$ and $\mathbf{\Sigma} \in \mathcal{C}^{m\times m}$ are the Hermitian positive-definite matrices

$$\mathbf{\Omega} \triangleq \begin{cases} \mathbf{R} & N_r \leqslant N_t \\ \mathbf{S} & N_r > N_t \end{cases}$$
$$\mathbf{\Sigma} \triangleq \begin{cases} \mathbf{S} & N_r \leqslant N_t \\ \mathbf{R} & N_r > N_t \end{cases} \quad (6)$$

with eigenvalues $\omega_1 < \ldots < \omega_n$ and $\sigma_1 < \ldots < \sigma_m$, respectively.

III. Maximum Eigenvalue Distribution of a Quadratic Form in Complex Gaussian Matrices

We now present a theorem that gives new first-order expansions for the p.d.f. and c.d.f. of the maximum eigenvalue of a quadratic form in complex Gaussian matrices. This will allow us to derive the asymptotic distribution of the SNR in (5).

Theorem 1: The following two expressions are first-order expansions of the c.d.f. and p.d.f., respectively, of the maximum eigenvalue $\lambda_{\max}$ of $\mathbf{Y} \sim \mathcal{Q}_{n,m}(\mathbf{I}_n, \mathbf{\Omega}, \mathbf{\Sigma})$:

$$F_{\lambda_{\max}}(x) = \alpha x^{mn} + o(x^{mn}) \quad (7)$$

and

$$f_{\lambda_{\max}}(x) = mn\alpha x^{mn-1} + o(x^{mn-1}) \quad (8)$$

where

$$\alpha = \frac{\Gamma_n(n)}{\det(\mathbf{\Omega})^m \det(\mathbf{\Sigma})^n \Gamma_n(m+n)} \quad (9)$$

and $\Gamma.(\cdot)$ is the normalized complex multivariate gamma function defined as

$$\Gamma_n(m) = \prod_{i=1}^{n} \Gamma(m-i+1). \quad (10)$$

Proof: The exact c.d.f. of the maximum eigenvalue of a positive-definite quadratic form in complex Gaussian matrices is given by ([11, Theorem 1])

$$F_{\lambda_{\max}}(x) = \frac{(-1)^n \Gamma_n(n) \det(\mathbf{\Omega})^{n-1} \det(\mathbf{\Sigma})^{m-1} \det(\mathbf{\Psi}(x))}{\Delta_n(\mathbf{\Omega}) \Delta_m(\mathbf{\Sigma}) (-x)^{n(n-1)/2}} \quad (11)$$

where $\Delta_m(\cdot)$ is a Vandermonde determinant in the eigenvalues of the m-dimensional matrix argument, given by

$$\Delta_m(\mathbf{\Sigma}) = \left|\sigma_i^{j-1}\right| = \prod_{i<j} (\sigma_j - \sigma_i). \quad (12)$$

Also, $\mathbf{\Psi}(x)$ is an $m \times m$ matrix with (i,j)th element

$$\{\mathbf{\Psi}(x)\}_{i,j} = \begin{cases} \left(\frac{1}{\sigma_j}\right)^{m-i}, & i \leqslant \tau \\ e^{-\frac{x}{\omega_{i-\tau}\sigma_j}} P(m; -\frac{x}{\omega_{i-\tau}\sigma_j}), & i > \tau \end{cases} \quad (13)$$

where $\tau = m - n$, and

$$P(l; y) = 1 - e^{-y} \sum_{k=0}^{l-1} y^k / k! \quad (14)$$

is the regularized lower incomplete gamma function.

We seek a first-order expansion of (11) that will be immediate from a first-order expansion of $\det(\mathbf{\Psi}(x))$. Consider the Taylor expansion of $\det(\mathbf{\Psi}(x))$ around the origin

$$\det(\mathbf{\Psi}(x)) = F(x) = \sum_{q=0}^{Q} \frac{F^{(q)}(0)}{q!} x^q + o(x^Q). \quad (15)$$

For a first-order expansion, we need to find the first nonzero coefficient in the sum. Using a well-known result for the qth derivative of a determinant, we have

$$F^{(q)}(0) = \sum_{\{q_1,\ldots,q_m\}} \frac{q!}{q_1! \cdots q_m!} \det\left(\frac{d^{q_i}\{\mathbf{\Psi}(x)\}_{i,j}}{dx^{q_i}}\right)\Bigg|_{x=0} \quad (16)$$

where $q_1 + \cdots + q_m = q$. Using (13) and (14), we evaluate the derivatives in (16) as follows:

$$\frac{d^{q_i}\{\mathbf{\Psi}(x)\}_{i,j}}{dx^{q_i}}\Bigg|_{x=0} = \begin{cases} \left(\frac{1}{\sigma_j^{m-i}}\right), & i \leqslant \tau \text{ and } q_i = 0 \\ 0, & i \leqslant \tau \text{ and } q_i > 0 \\ 0, & i > \tau \text{ and } 0 \leq q_i < m \\ \left(-\frac{1}{\omega_{i-\tau}\sigma_j}\right)^{q_i}, & i > \tau \text{ and } q_i \geq m. \end{cases} \quad (17)$$

From (17), in order for the determinants in (16) to be nonzero, we require that for rows $i \leqslant \tau$, we must have $q_i = 0$. We also require that for rows $i > \tau$, we must have $q_i \geq m$, and these q_i's must all be different. Hence, the *smallest* q for which these conditions are satisfied is given by

$$\begin{aligned} \tilde{q} &= m + (m+1) + \ldots + (m + (m - \tau - 1)) \\ &= mn + n(n-1)/2. \end{aligned} \quad (18)$$

For this smallest q value, we can now write (16) as follows:

$$F^{(\tilde{q})}(0) = \sum_{\{\underline{\alpha}\}} \frac{\tilde{q}!}{\Gamma_n(m+n)} \det\left(\frac{d^{\tilde{q}_i}\{\mathbf{\Psi}(x)\}_{i,j}}{dx^{\tilde{q}_i}}\right)\Bigg|_{x=0} \quad (19)$$

where the sum is over all permutations $\underline{\alpha} = \{\alpha_1, \ldots, \alpha_n\}$ of the numbers $\{0, \ldots, n-1\}$, and

$$\tilde{q}_i = \begin{cases} 0, & i \leqslant \tau \\ m + \alpha_{i-\tau}, & i > \tau. \end{cases} \quad (20)$$

Using (17) in (19), we have

$$F^{(\tilde{q})}(0) = \frac{\tilde{q}!(-1)^{\tilde{q}}}{\Gamma_n(m+n)} \sum_{\{\underline{\alpha}\}} \det(\mathbf{\Xi}_{\underline{\alpha}}) \quad (21)$$

where

$$\{\mathbf{\Xi}_{\underline{\alpha}}\}_{i,j} = \begin{cases} \left(\frac{1}{\sigma_j}\right)^{m-i}, & i \leqslant \tau \\ \left(\frac{1}{\omega_{i-\tau}\sigma_j}\right)^{m+\alpha_{i-\tau}}, & i > \tau. \end{cases} \quad (22)$$

We now focus on simplifying the determinant sum in (21). By removing the factors $(1/\omega_{i-\tau})^{m+\alpha_{i-\tau}}$ from the determinants and performing some row swaps, it can be shown that

$$\begin{aligned}\sum_{\{\underline{\alpha}\}} \det(\boldsymbol{\Xi}_{\underline{\alpha}}) &= \sum_{\{\underline{\alpha}\}} \left(\prod_{i=\tau+1}^{m} \left(\frac{1}{\omega_{i-\tau}}\right)^{m+\alpha_{i-\tau}} (-1)^{\mathrm{per}(\underline{\alpha})} \right. \\ &\quad \left. \times (-1)^{\tau(\tau-1)/2} \det\left(\left(\frac{1}{\sigma_i}\right)^{n+j-1}\right)\right) \\ &= \frac{(-1)^{\tau(\tau-1)/2}\det\left(\left(\frac{1}{\sigma_i}\right)^{j-1}\right)}{\det(\boldsymbol{\Omega})^m \det(\boldsymbol{\Sigma})^n} \\ &\quad \times \sum_{\{\underline{\alpha}\}} (-1)^{\mathrm{per}(\underline{\alpha})} \prod_{i=1}^{n} \left(\frac{1}{\omega_i}\right)^{\alpha_i} \\ &= \frac{(-1)^{\tau(\tau-1)/2}\det\left(\left(\frac{1}{\sigma_i}\right)^{j-1}\right)\det\left(\left(\frac{1}{\omega_i}\right)^{j-1}\right)}{\det(\boldsymbol{\Omega})^m \det(\boldsymbol{\Sigma})^n} \end{aligned} \tag{23}$$

where the last line followed from the definition of the determinant. Next, using (12) and the Vandermonde determinant identity [11, (56)]

$$\prod_{i<j}^{m} \left(\frac{1}{\sigma_j} - \frac{1}{\sigma_i}\right) = \frac{\prod_{i<j}^{m} (\sigma_i - \sigma_j)}{\prod_{i=1}^{m} \sigma_i^{m-1}} \tag{24}$$

we can write (23) as follows:

$$\begin{aligned}\sum_{\{\underline{\alpha}\}} \det(\boldsymbol{\Xi}_{\underline{\alpha}}) &= (-1)^{\tau(\tau-1)/2}(-1)^{m(m-1)/2}(-1)^{n(n-1)/2} \\ &\quad \times \frac{\Delta_m(\boldsymbol{\Sigma})\Delta_n(\boldsymbol{\Omega})}{\det(\boldsymbol{\Sigma})^{m+n-1}\det(\boldsymbol{\Omega})^{m+n-1}} \\ &= (-1)^{n(m+1)} \frac{\Delta_m(\boldsymbol{\Sigma})\Delta_n(\boldsymbol{\Omega})}{\det(\boldsymbol{\Sigma})^{m+n-1}\det(\boldsymbol{\Omega})^{m+n-1}}. \end{aligned} \tag{25}$$

Substituting (25) into (21) and using (18), we derive the desired first-order expansion of $\det(\boldsymbol{\Psi}(x))$ in (15), given by

$$\begin{aligned}\det(\boldsymbol{\Psi}(x)) &= \frac{(-1)^n(-1)^{n(n-1)/2}\Delta_m(\boldsymbol{\Sigma})\Delta_n(\boldsymbol{\Omega})}{\Gamma_n(m+n)\det(\boldsymbol{\Sigma})^{m+n-1}\det(\boldsymbol{\Omega})^{m+n-1}} \\ &\quad \times x^{mn+n(n-1)/2} + o(x^{mn+n(n-1)/2}). \end{aligned} \tag{26}$$

The c.d.f. result (7) now follows by substituting (26) into (11) and simplifying. The p.d.f. result (8) then follows trivially by taking the derivative of (7) w.r.t. x. □

For the special case of uncorrelated fading, (7) reduces to

$$F_{\lambda_{\max}}(x) = \frac{\Gamma_n(n)}{\Gamma_n(m+n)} x^{mn} + o(x^{mn}) \tag{27}$$

which agrees with a result derived previously in [7].

IV. Asymptotic SER and Outage Analysis of MIMO-MRC

For many general modulation formats, the average SER of MIMO-MRC can be expressed as

$$P_s = E_\gamma[aQ(\sqrt{2b\gamma})] \tag{28}$$

where $Q(\cdot)$ is the Gaussian Q-function, and a and b are modulation-specific constants [15]. In [16], an alternative expression of (28) was provided as follows:

$$P_s = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-bu}}{\sqrt{u}} F_{\lambda_{\max}}(u)\, du. \tag{29}$$

We now analyze the SER performance in the high SNR regime in order to derive the diversity order and array gain of the system. Armed with *Theorem 1*, we can directly invoke a general parameterized single-input single-output (SISO) SER result from [17], and perform some basic algebraic manipulations, to obtain a high SNR SER expression given by

$$\mathrm{SER}^\infty = (G_a \cdot \bar{\gamma})^{-G_d} + o(\bar{\gamma}^{-G_d}) \tag{30}$$

where the diversity order is

$$G_d = mn \tag{31}$$

and the array gain is

$$\begin{aligned} G_a &= \det(\boldsymbol{\Omega})^{1/n}\det(\boldsymbol{\Sigma})^{1/m} 2b \\ &\quad \times \left(\frac{a\Gamma_n(n)}{2\Gamma_n(m+n)}(2mn-1)!!\right)^{-1/mn} \end{aligned} \tag{32}$$

with

$$(2mn-1)!! \triangleq 1 \times 3 \times \cdots \times (2mn-1). \tag{33}$$

We clearly see that MIMO-MRC achieves the full spatial diversity order of mn, regardless of the spatial correlation. Moreover, using Hadamard's inequality and the fact that the diagonal elements of $\boldsymbol{\Omega}$ and $\boldsymbol{\Sigma}$ are unity, it is easily found that

$$0 \leqslant \det(\boldsymbol{\Omega}) \leqslant 1 \quad \text{and} \quad 0 \leqslant \det(\boldsymbol{\Sigma}) \leqslant 1 \tag{34}$$

with equality in the upper limit only when the correlation matrices are identity matrices. Hence, from (32), we see that the effect of the correlation is to reduce the array gain (with respect to uncorrelated fading) by a factor of $\det(\boldsymbol{\Omega})^{1/n}\det(\boldsymbol{\Sigma})^{1/m}$, thereby increasing the SER in the high SNR regime. Note that for the special case $n = 2$, this result can be shown to reduce to an expression reported previously in [11].

We now consider the outage probability of MIMO-MRC systems in double-correlated Rayleigh channels. The outage probability is an important quality of service measure, defined as the probability that γ drops below an acceptable SNR threshold γ_{th}. It is obtained using (5) as follows:

$$F_\gamma(\gamma_{\mathrm{th}}) = \Pr(\gamma \leqslant \gamma_{\mathrm{th}}) = F_{\lambda_{\max}}\left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}\right). \tag{35}$$

In practice, we are usually interested in small outage probabilities (i.e., 0.01, 0.001, ...), which correspond to small values of γ_{th}. To gain further intuition into these small outage probabilities, we use (7) in *Theorem 1* to write the outage probability in (35) as follows:

$$\begin{aligned}\tilde{F}_\gamma(\gamma_{\mathrm{th}}) &= \frac{\Gamma_n(n)}{\det(\boldsymbol{\Omega})^m\det(\boldsymbol{\Sigma})^n\Gamma_n(m+n)} \\ &\quad \times \left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}\right)^{mn} + o((\gamma_{\mathrm{th}})^{mn}). \end{aligned} \tag{36}$$

According to (34), this result shows explicitly that in the low outage regime, the outage performance degrades due to the presence of spatial correlation. Moreover, the increase in outage

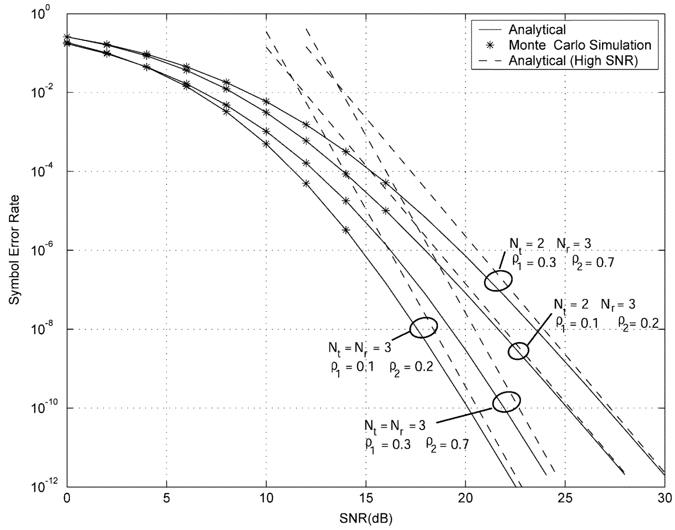

Fig. 1. SER of MIMO-MRC in various double-correlated Rayleigh channels with 8PSK modulation.

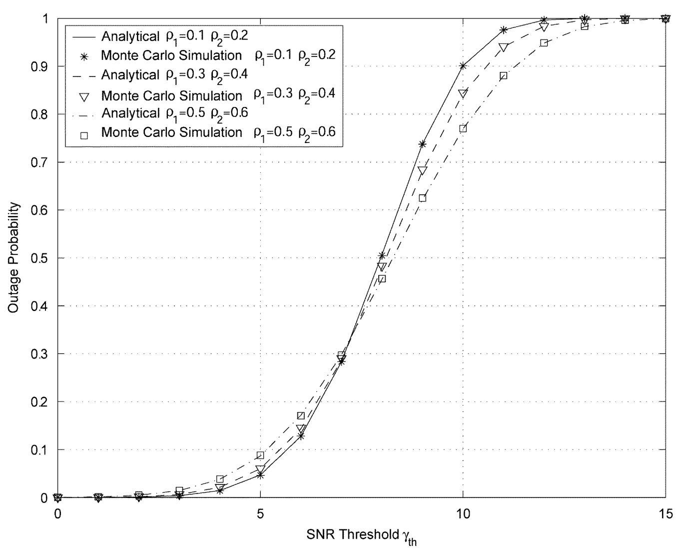

Fig. 2. Outage probability of 3×3 MIMO-MRC in various double-correlated Rayleigh channels and for $\bar{\gamma} = 0$ dB.

probability (with respect to uncorrelated fading) is quantified by the factor $\det(\mathbf{\Omega})^{-m} \det(\mathbf{\Sigma})^{-n}$.

V. Numerical Results

For our numerical results, we construct the correlation matrices using the exponential correlation model. The (i, j)th entries of $\mathbf{\Omega}$ and $\mathbf{\Sigma}$ are given by $\{\mathbf{\Omega}\}_{i,j} = \rho_1^{|i-j|}$ and $\{\mathbf{\Sigma}\}_{i,j} = \rho_2^{|i-j|}$ with $\rho_1, \rho_2 \in [0, 1)$, respectively. Note however, that all of the analytical results presented in this letter apply equally to any other correlation model that conforms to the general structure in (2).

Fig. 1 shows the SER of MIMO-MRC with 8PSK $(a = 2, b = 0.146)$ modulation, for various antenna configurations. The "Analytical" curves are generated via numerical integration of (29) using the analytical c.d.f. (11). The "Analytical (High SNR)" curves are based on (30). Clearly, the diversity orders and array gains predicted by the high SNR analytical results are accurate. Also, as expected from the analysis in Section IV, we see that the SER increases monotonically with the level of correlation for both antenna configurations.

Fig. 2 shows analytical and Monte-Carlo simulation outage probability curves for a MIMO-MRC system, comparing different correlation scenarios. The analytical results are based on (11). As expected from our asymptotic analysis in Section IV, we see that the correlation increases the outage probability for all outage levels of practical interest (i.e., in this case, for outage levels $<30\%$). It is also interesting to observe that the opposite occurs for high outage levels.

VI. Conclusion

We have examined the asymptotic performance of MIMO-MRC systems in double-correlated Rayleigh channels. Our results are based on new closed-form asymptotic expressions, which we have derived for the marginal maximum eigenvalue distribution of positive-definite quadratic forms in complex Gaussian matrices. The new results prove that the presence of spatial correlation yields a net increase in SER in the high SNR regime and also degrades the outage performance for outage levels of practical interest.

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