Transmit Antenna Selection Schemes with Reduced Feedback Rate

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***Abstract*— In this paper, we propose and analyze three new transmit antenna selection schemes with reduced feedback rate requirement compared with the conventional scheme. In Scheme 1, L_t available transmit antennas are divided as equally as possible into two groups with consecutive antennas. The best single antenna within each group is selected. In Scheme 2, only the best one among L_t antennas is made known to the transmitter, and the other one is selected at random. In Scheme 3, L_t antennas are divided into multiple subsets each consisting of two adjacent antennas, and the best subset is selected. Bit error rate (BER) expressions for the proposed schemes with Alamouti code are derived for independent flat Rayleigh fading channels. It is found that all the three schemes achieve a full diversity order. The relative merit of each proposed scheme is delineated based on the trade-off between the asymptotic performance loss and feedback reduction, both relative to the conventional scheme. We conclude that Schemes 1 and 3 are more favorable for practical applications, and the appropriate application scenarios are also identified. The proposed schemes enrich the choices for antenna selection system design for various feedback channel bandwidths and different requirements for quality of service.**

***Index Terms*— Antenna selection, diversity, fading channels, space-time block code.**

I. INTRODUCTION

MULTIPLE-input-multiple-output (MIMO) systems with antenna subset selection for space-time codes have been a subject of intense research [1], [2]. In general, antenna subset selection can be classified as receive antenna selection and transmit antenna selection. Receive antenna selection [3]–[8] reduces the number of required radio-frequency (RF) chains at the receiver, which leads to significant cost savings. The savings come at the cost of a usually small performance loss compared to the full-complexity system without antenna selection. The idea behind transmit antenna selection is that a subset of the transmit antennas are selected for transmission. Usually space-time codes designed for a small number of antennas are employed. Transmit antenna selection with space-time block code (TAS/STBC) [9]–[12] and space-time trellis code (TAS/STTC) [13], [14] have been investigated extensively. In addition to reducing the number of RF chains at the transmitter, transmit antenna selection can keep the complexity of mobile sets to minimum by deploying most of the antennas at the base station in downlink transmission. However, a feedback link has to be established for the scenarios where channel state information (CSI) cannot be estimated at the transmitter, such as frequency division duplexing.

For the conventional transmit antenna selection schemes with space-time codes, such as TAS/STBC and TAS/STTC in [2], [9], [13] and [14], the antennas within the selected subset are globally optimal in the sense that they are selected among all the available antennas without restriction. The selected subset usually consists of two antennas due to code availability and superiority of error performance. The corresponding number of feedback bits for the system with L_t transmit antennas is $\lceil \log_2 \binom{L_t}{2} \rceil$, where $\lceil x \rceil$ denotes the smallest integer not smaller than x. Most of the practical wireless communications systems can only provide a feedback channel with a limited bandwidth. For example, a feedback rate of 1.5 k bits/s is allocated for the third generation (3G) cellular systems [15]. Therefore, it would be desirable to develop new schemes which can further reduce the feedback rate requirement without significantly compromising the error performance compared with the conventional scheme.

With this objective, we propose three transmit antenna selection schemes in this paper, denoted by Schemes 1-3. These three schemes all select two out of L_t available antennas for transmission. In Scheme 1, L_t antennas are divided into two groups with consecutive antennas equally if L_t is an even number, or as equally as possible if L_t is an odd number. The antenna corresponding to the maximal channel power gain within each of the two groups is selected. In Scheme 2, only the index of the best one among L_t antennas is fed back to the transmitter, and the other antenna is chosen at random. In Scheme 3, L_t transmit antennas are divided into subsets each of which has two adjacent antennas. If L_t is even, there are $L_t/2$ subsets without antenna overlapping. If L_t is odd, there are $(L_t+1)/2$ subsets. The first $(L_t-3)/2$ subsets have no common antenna, and the last two subsets overlap with a common antenna. The subset with the largest instantaneous channel power gain will be selected.

It is clear that all the three proposed schemes place certain restrictions on the antenna subset selection, which is not globally optimal compared with the conventional scheme. Therefore, the reduction in feedback rate is achieved at the cost of performance degradation. From a wireless system

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engineer's viewpoint, we would like to analyze the error performances of these three schemes, quantify the feedback reduction, and determine which scheme gives the best trade-off between performance and feedback requirement for different application scenarios. In the error performance analysis, it is assumed that the binary phase-shift keying (BPSK) Alamouti STBC [16] is employed in all the three proposed schemes. Taking into account the fact that most hand-held devices only have a single antenna in a cellular radio environment, we mainly focus on the scenario with a single receive antenna in the performance analysis. It is found that all the three proposed schemes achieve a full diversity order in independent flat Rayleigh fading channels, as if all the transmit antennas were used. In general, these three proposed schemes provide a good trade-off between error performance and feedback requirement in different application scenarios. Compared with the conventional antenna selection scheme, Scheme 1 achieves a feedback rate reduction by 1 bit for most L_t at a negligible performance loss. Scheme 3 can reduce feedback rate by about half with a performance loss upper bounded by 1.5 dB at large L_t when compared with the conventional scheme. However, Scheme 2 always needs one more feedback bit than Scheme 3 with a performance loss upper bounded by 3 dB at large L_t relative to the conventional scheme. Therefore, we reach the conclusion that Schemes 1 and 3 are more favorable in practical applications.

The major contribution of this paper can be summarized as: 1) the proposal of three novel transmit antenna selection schemes with reduced feedback rate requirement; 2) the error performance analysis of the proposed schemes; and 3) the identification of the relative merits and application scenarios for each scheme.

The rest of the paper is organized as follows. Section II briefly introduces the channel model and describes the three proposed schemes. Section III quantifies the asymptotic conditional bit error rate (BER) of a transmit antenna selection system where two selected antennas have fixed ordinal numbers. Based on the intermediate results in Section III, the asymptotic BER expressions for Schemes 1 and 2 are derived in Section IV. In addition, the error performance of Scheme 3 is obtained by the moment generating function (MGF) method [17]. In Section V, the trade-off between performance and feedback rate for the proposed schemes is evaluated, and the appropriate application scenarios are identified. Numerical results are provided in Section VI to substantiate the analysis. Concluding remarks are given in Section VII.

II. Channel Model and Description of the Proposed Schemes

We consider a MIMO system equipped with $L_t \geq 2$ transmit and L_r receive antennas. The fading coefficients $h_{j,i}$ between transmit antenna i and receive antenna j, $1 \leq i \leq L_t, 1 \leq j \leq L_r$, are independent identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables of zero mean and unit variance. We define

$$C_i = \sum_{j=1}^{L_r} |h_{j,i}|^2, \quad 1 \leq i \leq L_t, \tag{1}$$

which is the instantaneous channel power gain between transmit antenna i and all the receive antennas. We rearrange the random variables C_i in ascending order of magnitude and denote them by $C_{(l)}$, where $1 \leq l \leq L_t$ and $C_{(1)} \leq C_{(2)} \leq \cdots \leq C_{(L_t)}$. l is referred to as the ordinal number of the antenna associated with $C_{(l)}$. The transmit antenna associated with $C_{(L_t)}$ is referred to as the best antenna among L_t antennas.

In this paper, we only consider $(L_t, 2; L_r)$ transmit antenna selection schemes, where two out of L_t antennas are selected for transmission, and all the L_r receive antennas are used.

The conventional transmit antenna selection schemes in [2], [9], [11] and [13] select the best two antennas globally without restriction. The two antennas, denoted by U and V, are determined by

$$\{U, V\} = \underset{1\leq u,v\leq L_t, u\neq v}{\text{argmax}} \{C_u + C_v\}, \tag{2}$$

which means that the two antennas associated with $C_{(L_t)}$ and $C_{(L_t-1)}$ are selected. The index of the subset consisting of U and V is fed back to the transmitter, and the number of required feedback bits, denoted by N, is determined by

$$N = \lceil \log_2 \binom{L_t}{2} \rceil. \tag{3}$$

The justification of N is included in Appendix I-A. Next, we will propose three different antenna selection schemes with certain antenna selection restrictions to reduce the number of feedback bits.

A. Scheme 1

The block diagram of Scheme 1 is shown in Fig. 1. L_t antennas are divided into two groups with N_{G_1} and N_{G_2} consecutive antennas in Group 1 and Group 2, respectively. N_{G_1} and N_{G_2} are calculated as

$$\begin{aligned} &N_{G_1} = N_{G_2} = \frac{L_t}{2}, && L_t \text{ is even} \\ &N_{G_1} = \frac{L_t - 1}{2}, \ N_{G_2} = \frac{L_t + 1}{2}, && L_t \text{ is odd.} \end{aligned} \tag{4}$$

For example, if $L_t = 7$, Group 1 will include three consecutive antennas (1,2,3), and Group 2 will have the remaining four consecutive antennas (4,5,6,7).

The two selected antennas are determined by

$$\{U, V\} = \underset{1\leq u\leq N_{G_1}, N_{G_1}+1\leq v\leq L_t}{\text{argmax}} \{C_u + C_v\}. \tag{5}$$

The antenna with the largest ordinal number within each of the two antenna groups is selected to form a subset with two antennas. Selection criterion (5) guarantees that the best one among L_t antennas is always within the selected subset.

Following Appendix I-B, we have the number of feedback bits as

$$N_1 = \begin{cases} \lceil 2\log_2 L_t - 2 \rceil, & L_t \text{ is even} \\ \lceil \log_2 (L_t - 1) + \log_2 (L_t + 1) - 2 \rceil, & L_t \text{ is odd.} \end{cases} \tag{6}$$

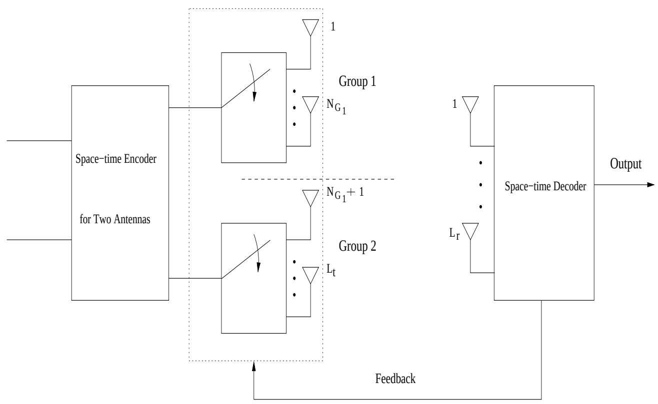

Fig. 1. The block diagram of Scheme 1.

B. Scheme 2

In Scheme 2, only the index of the best antenna, determined by

$$U = \underset{1\le u\le L_t}{\operatorname{argmax}}\left\{C_u\right\}, \tag{7}$$

is fed back to the transmitter, and the other antenna is selected at random. Selection criterion (7) also guarantees that the best antenna is always within the selected subset. The number of feedback bits is given by

$$N_2 = \lceil \log_2 L_t \rceil. \tag{8}$$

C. Scheme 3

In Scheme 3, transmit antennas are divided into subsets each of which has two adjacent antennas.

If L_t is even, L_t antennas are divided into $\frac{L_t}{2}$ subsets. For example, if $L_t = 6$, the three subsets are formed as (1,2), (3,4), and (5,6). The selected subset, denoted by I, is determined by

$$I = \underset{1\le i\le \frac{L_t}{2}}{\operatorname{argmax}}\left\{C_{2i-1}+C_{2i}\right\}, \tag{9}$$

and the two selected antennas are

$$U = 2I-1, \quad V = 2I. \tag{10}$$

A special case of Scheme 3 with $L_t = 4$ first appeared in [18].

If L_t is odd, $\frac{L_t+1}{2}$ subsets will be formed. The first $L_t - 1$ antennas are divided into $\frac{L_t-1}{2}$ subsets. The last antenna will form a subset together with the adjacent antenna, which also belongs to the adjacent subset. For example, if $L_t = 5$, the three subsets will be formed as (1,2), (3,4), and (4,5). The selected subset is determined by

$$I = \underset{1\le i\le \frac{L_t+1}{2}}{\operatorname{argmax}}\left\{\underset{1\le i\le \frac{L_t-1}{2}}{C_{2i-1}+C_{2i}}, \underset{i=\frac{L_t+1}{2}}{C_{2i-2}+C_{2i-1}}\right\}, \tag{11}$$

and the two selected antennas are

$$\begin{cases} U = 2I-1, \quad V = 2I, & 1 \le I \le \frac{L_t-1}{2} \\ U = L_t-1, \quad V = L_t, & I = \frac{L_t+1}{2}. \end{cases} \tag{12}$$

Note that selection criteria (9) and (11) unnecessarily guarantee that the best one among L_t antennas is always within the selected subset.

The number of feedback bits of Scheme 3 is calculated as

$$N_3 = \begin{cases} \lceil \log_2 \frac{L_t}{2} \rceil = \lceil \log_2 L_t \rceil - 1, & L_t \text{ even} \\ \lceil \log_2 \frac{L_t+1}{2} \rceil = \lceil \log_2 (L_t+1) \rceil - 1, & L_t \text{ odd.} \end{cases} \tag{13}$$

Taking into account that $\lceil \log_2 (L_t+1)\rceil = \lceil \log_2 L_t \rceil$ if L_t is odd, we can write (13) as

$$N_3 = N_2 - 1, \tag{14}$$

which indicates that Scheme 3 always needs one bit less than Scheme 2.

Assuming that BPSK Alamouti STBC [16] is employed, next we will investigate the error performances of these three proposed schemes with $L_r = 1$ in independent flat Rayleigh fading channels. It is also assumed that perfect CSI is available to the receiver, and that there is no feedback delay or error.

III. Conditional Asymptotic bit error performance

In this section we will quantify the conditional asymptotic error performance of a transmit antenna selection system with BPSK Alamouti STBC with subset $\{(n),(m)\}$, $1 \le m < n \le L_t$. $\{(n),(m)\}$ denotes that the subset consists of the two antennas associated with $C_{(n)}$ and $C_{(m)}$. The intermediate results obtained will be utilized in Section IV for the derivation of average error expressions for Scheme 1 and Scheme 2.

It is shown in [19] that at high SNRs, the error performance of wireless transmission systems over fading channels can be calculated based on the behavior of the probability density function (pdf) of the instantaneous channel power gain, denoted by β. Assume that β-dependent instantaneous symbol error probability is given by $P_E(\beta) = Q\left(\sqrt{k\beta\bar{\gamma}}\right)$, where k is a positive fixed constant with $k = 2$ for BPSK, and $\bar{\gamma} = \frac{E_b}{N_0}$, where E_b is the energy per bit at the transmitter, and N_0 is the power spectral density of the additive white Gaussian noise (AWGN) per receive antenna.

We write $f(x) = o\left[g(x)\right]$, $x \to x_0$, if $\lim_{x\to x_0} \frac{f(x)}{g(x)} = 0$. Following [19], if the pdf of β, denoted by $p_\beta(\beta)$, can be approximated by a single polynomial term for $\beta \to 0^+$ as

$$p_\beta(\beta) = a\beta^t + o(\beta^t), \quad a > 0, \tag{15}$$

then the asymptotic symbol error probability is given by [19]

$$P_E = \frac{2^t\, a\,\Gamma(t+3/2)}{\sqrt{\pi}(t+1)} \cdot (k\bar{\gamma})^{-(t+1)} + o(\bar{\gamma}^{-(t+1)}), \tag{16}$$

where $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx$ is the Gamma function, and $t+1$ is the system diversity order. Throughout the paper, asymptotic means that SNR tends to infinity ($\bar{\gamma} \to \infty$).

Let $Y_1 = C_{(m)}$ and $Y_0 = C_{(n)}$. The post-processing SNR of the transmit antenna selection system with BPSK Alamouti STBC can be written as

$$\gamma = \frac{\bar{\gamma}}{2}\left[Y_1 + Y_0\right] = \frac{\bar{\gamma}}{2} Y. \tag{17}$$

We have the instantaneous channel power gain as $\beta = \frac{Y}{2}$.

For $L_r = 1$, C_i in (1) are i.i.d. chi-squared variables with 2 degrees of freedom. The pdf of C_i is given by [20, (2-1-110)]

$$p_{C_i}(y) = e^{-y}, \quad y \ge 0, \tag{18}$$

and the cumulative distribution function (cdf) of C_i is expressed as [20, (2-1-114)]

$$P_{C_i}(y) = 1 - e^{-y}, \quad y \geq 0. \tag{19}$$

Following the methodology in [21], the joint pdf of Y_0 and Y_1 is given by

$$\begin{aligned} f_{Y_0Y_1} &= \frac{L_t!}{(m-1)!(n-m-1)!(L_t-n)!} P_{C_i}^{m-1}(y_1) p_{C_i}(y_1) \\ &\quad \times \left[P_{C_i}(y_0) - P_{C_i}(y_1)\right]^{n-m-1} p_{C_i}(y_0) \left[1 - P_{C_i}(y_0)\right]^{L_t-n} \\ &= \frac{L_t!}{(m-1)!(n-m-1)!(L_t-n)!} \\ &\quad \times \sum_{t=0}^{m-1} \sum_{s=0}^{n-m-1} \binom{m-1}{t} \binom{n-m-1}{s} \\ &\quad \times (-1)^{s+t} e^{-(L_t+s-n+1)y_0} e^{-(n+t-m-s)y_1}. \end{aligned} \tag{20}$$

For $Y = Y_0 + Y_1$, we have

$$\begin{aligned} &P_Y(y) \\ &= \Pr\{Y \leq y\} \\ &= \int_0^{y/2} \int_{y_1}^{y-y_1} f_{Y_0Y_1}(y_0, y_1) dy_0 dy_1 \\ &= \frac{L_t!}{(m-1)!(n-m-1)!(L_t-n)!} \\ &\quad \times \sum_{t=0}^{m-1} \sum_{s=0}^{n-m-1} (-1)^{s+t} \binom{m-1}{t} \binom{n-m-1}{s} \\ &\quad \times \int_0^{y/2} \int_{y_1}^{y-y_1} e^{-(L_t+s-n+1)y_0} e^{-(n+t-m-s)y_1} dy_0 dy_1 \\ &= \frac{L_t!}{(m-1)!(n-m-1)!(L_t-n)!} \\ &\quad \times \Bigg\{ \underbrace{\sum_{t=0}^{m-1} \sum_{s=0}^{n-m-1}}_{t \neq 2s+(L_t+m-2n+1)} \frac{(-1)^{s+t+1} \binom{m-1}{t} \binom{n-m-1}{s}}{L_t+s-n+1} \\ &\qquad \times \Bigg[\frac{e^{-\frac{1}{2}(L_t-m+t+1)y} - e^{-(L_t+s-n+t)y}}{L_t+2s+m-2n-t+1} \\ &\qquad\qquad + \frac{e^{-\frac{1}{2}(L_t+t+1-m)y} - 1}{L_t+t+1-m} \Bigg] \\ &\quad + \underbrace{\sum_{t=0}^{m-1} \sum_{s=0}^{n-m-1}}_{t = 2s+(L_t+m-2n+1)} \frac{(-1)^{s+t+1} \binom{m-1}{t} \binom{n-m-1}{s}}{L_t+s-n+1} \\ &\qquad \times \left[\frac{y}{2} e^{-(L_t+s-n+1)y} + \frac{e^{-\frac{1}{2}(L_t+t+1-m)y} - 1}{L_t+t+1-m} \right] \Bigg\}. \end{aligned} \tag{21}$$

From (21), we have the pdf of Y as

$$\begin{aligned} p_Y(y) &= \frac{dP_Y(y)}{dy} \\ &= \frac{L_t!}{(m-1)!(n-m-1)!(L_t-n)!} \\ &\quad \times \Bigg\{ \underbrace{\sum_{t=0}^{m-1} \sum_{s=0}^{n-m-1}}_{t \neq 2s+(L_t+m-2n+1)} \frac{(-1)^{s+t} \binom{m-1}{t} \binom{n-m-1}{s}}{L_t+2s+m-2n-t+1} \\ &\qquad \times \left[e^{-\frac{1}{2}(L_t-m+t+1)y} - e^{-(L_t+s-n+1)y} \right] \\ &\qquad + \frac{1}{2} \underbrace{\sum_{t=0}^{m-1} \sum_{s=0}^{n-m-1}}_{t = 2s+(L_t+m-2n+1)} (-1)^{s+t} \binom{m-1}{t} \binom{n-m-1}{s} \\ &\qquad\qquad \times y e^{-(L_t+s-n+1)y} \Bigg\}. \end{aligned} \tag{22}$$

Since $\beta = \frac{Y}{2}$, we have

$$\begin{aligned} p_\beta(\beta) &= 2p_Y(2\beta) \\ &= \frac{2L_t!}{(m-1)!(n-m-1)!(L_t-n)!} \\ &\quad \times \Bigg\{ \underbrace{\sum_{t=0}^{m-1} \sum_{s=0}^{n-m-1}}_{t \neq 2s+(L_t+m-2n+1)} \frac{(-1)^{s+t} \binom{m-1}{t} \binom{n-m-1}{s}}{L_t+2s+m-2n-t+1} \\ &\qquad \times \left[e^{-(L_t-m+t+1)\beta} - e^{-2(L_t+s-n+1)\beta} \right] \\ &\qquad + \underbrace{\sum_{t=0}^{m-1} \sum_{s=0}^{n-m-1}}_{t = 2s+(L_t+m-2n+1)} (-1)^{s+t} \binom{m-1}{t} \binom{n-m-1}{s} \\ &\qquad\qquad \times \beta e^{-2(L_t+s-n+1)\beta} \Bigg\}. \end{aligned} \tag{23}$$

In order to obtain the asymptotic error probability, (23) should be written in the form of (15). If $\beta \to 0$, (23) can be written as

$$p_\beta(\beta) = \frac{2^{n-m} L_t!}{(n-1)!(L_t-n)!} \beta^{n-1} + o\left(\beta^{n-1}\right). \tag{24}$$

The comparison between (24) and (15) reveals that

$$a = \frac{2^{n-m} L_t!}{(n-1)!(L_t-n)!} \tag{25}$$

$$t = n-1. \tag{26}$$

Substituting (25) and (26) into (16), we obtain the conditional asymptotic BER expression for a transmit antenna selection system with BPSK Alamouti STBC and subset $\{(n), (m)\}$ for a single receive antenna, denoted by $P_2(n, m)$, as

$$P_2(n, m) = \frac{L_t!(2n-1)!}{2^{n+m} n!(n-1)!(L_t-n)!} \bar{\gamma}^{-n} + o\left(\bar{\gamma}^{-n}\right) \tag{27}$$

at high SNRs.

Equation (27) clearly indicates that the asymptotic diversity order is n, the larger ordinal number of the antenna within

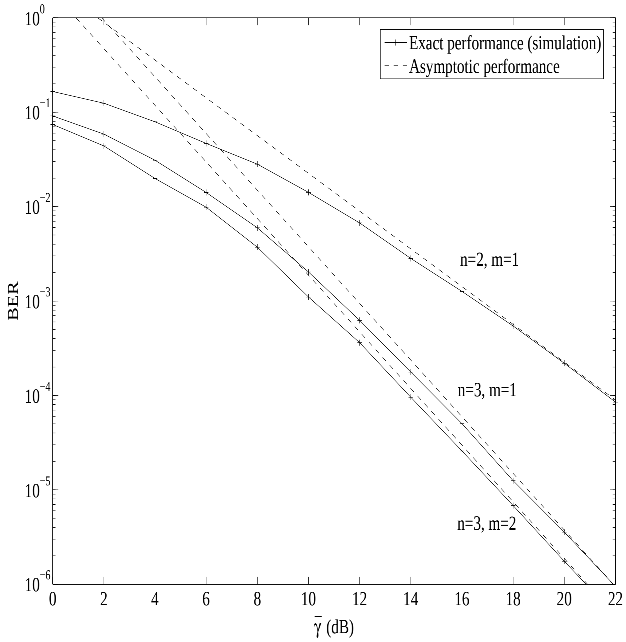

Fig. 2. Performance comparison of the (3,2;1) TAS/STBC with different subsets selected, BPSK.

the subset. The smaller ordinal number, m, only determines the horizontal location of the error performance curve, or the SNR gain. For a fixed n, every increase by one in m brings an asymptotic SNR advantage of $\frac{3}{n}$ dB.

In order to validate the analysis in this section, Fig. 2 compares the BER performance by simulation with the asymptotic expression (27) for the (3,2;1) BPSK TAS/STBC with all the possible antenna subsets, $\{(2),(1)\}$, $\{(3),(1)\}$, and $\{(3),(2)\}$. It is shown that the asymptotic diversity order is two for subset $\{(2),(1)\}$, and three for both scenarios with $\{(3),(1)\}$, and $\{(3),(2)\}$. The asymptotic SNR advantage of $\{(3),(2)\}$ over $\{(3),(1)\}$ is 1 dB at the BER of 10^{-6}, the same as predicted by (27). Fig. 2 also indicates that (27) asymptotically approaches the exact error performance.

In practice, it is rare that the values of the ordinal numbers, n and m, are always fixed. However, the conditional BER results in this section serve as important intermediate results in the derivation of the average BER expressions for Schemes 1 and 2.

IV. Bit Error Performance of the Proposed Schemes

In this section, we will derive the asymptotic BER expressions for the three proposed schemes with BPSK Alamouti STBC and a single receive antenna. The error performance analysis for Schemes 1 and 2 is based on the intermediate results in Section III, and that for Scheme 3 employs the MGF method.

A. Scheme 1

For Scheme 1, we have $n = L_t$. Therefore, the index n in (27) can be replaced by L_t and dropped. Accordingly, we have

$$P_2(m) = P_2(n = L_t, m) = \frac{(2L_t-1)!}{2^{m+L_t}(L_t-1)!}\bar{\gamma}^{-L_t} + o\left(\bar{\gamma}^{-L_t}\right). \tag{28}$$

Let $P(m)$ define the probability that the ordinal number of the second selected antenna is m. To obtain the unconditional average BER, $P_2(m)$ should be averaged over $P(m)$. We will differentiate between the cases of L_t even and L_t odd.

1) L_t is an Even Number: If L_t is even, Group 1 and Group 2 will each contain $\frac{L_t}{2}$ antennas. We have

$$P(m) = \frac{\binom{m-1}{\frac{L_t}{2}-1}\left(\frac{L_t}{2}\right)!\left(\frac{L_t}{2}-1\right)!}{(L_t-1)!}, \quad \frac{L_t}{2} \le m \le L_t - 1. \tag{29}$$

Then the BER expression can be written as

$$P_2 = \sum_{m=\frac{L_t}{2}}^{L_t-1} P_2(m)P(m). \tag{30}$$

Substituting (28) and (29) into (30), we obtain

$$P_2 = \sum_{m=\frac{L_t}{2}}^{L_t-1} \frac{\binom{m-1}{\frac{L_t}{2}-1}}{2^m}\frac{(2L_t-1)!\left(\frac{L_t}{2}\right)!\left(\frac{L_t}{2}-1\right)!}{2^{L_t}\left[(L_t-1)!\right]^2}\bar{\gamma}^{-L_t} + o\left(\bar{\gamma}^{-L_t}\right)$$
$$= \frac{(2L_t-1)!\left(\frac{L_t}{2}\right)!\left(\frac{L_t}{2}-1\right)!}{2^{L_t+1}\left[(L_t-1)!\right]^2}\bar{\gamma}^{-L_t} + o\left(\bar{\gamma}^{-L_t}\right). \tag{31}$$

Note that

$$\sum_{m=\frac{L_t}{2}}^{L_t-1} \frac{\binom{m-1}{\frac{L_t}{2}-1}}{2^m} = \frac{1}{2} \tag{32}$$

is utilized in the derivation of (31).

For the conventional $(L_t, 2; 1)$ TAS/STBC scheme with BPSK modulation, at high SNRs, the BER can be written as [9, (7)]

$$P_2 = \frac{(2L_t-1)!}{2^{2L_t-1}(L_t-1)!}\bar{\gamma}^{-L_t} + o\left(\bar{\gamma}^{-L_t}\right). \tag{33}$$

The comparison between (31) and (33) indicates that Scheme 1, when compared with the conventional $(L_t, 2; 1)$ TAS/STBC scheme, suffers an asymptotic SNR loss given by

$$\Delta_1 = \frac{10}{L_t}\log_{10}\frac{2^{L_t-1}\left[\left(\frac{L_t}{2}\right)!\right]^2}{L_t!} \quad \text{dB}, \tag{34}$$

if L_t is an even number.

2) L_t is an Odd Number: If L_t is odd, Group 1 and Group 2 will consist of $\frac{L_t-1}{2}$ and $\frac{L_t+1}{2}$ antennas, respectively. Then

$$P(m) = \frac{\binom{m-1}{\frac{L_t-1}{2}}\left(\frac{L_t+1}{2}\right)!\left(\frac{L_t-1}{2}\right)! + \binom{m-1}{\frac{L_t-3}{2}}\left(\frac{L_t-1}{2}\right)!\left(\frac{L_t+1}{2}\right)!}{L_t!}, \tag{35}$$

where $\frac{L_t-1}{2} \le m \le L_t - 1$. Considering the identity

$$\binom{p}{q} + \binom{p}{q+1} = \binom{p+1}{q+1}, \tag{36}$$

we can simplify (35) as

$$P(m) = \frac{\binom{m}{\frac{L_t-1}{2}}\left(\frac{L_t+1}{2}\right)!\left(\frac{L_t-1}{2}\right)!}{L_t!}. \tag{37}$$

Therefore, the BER can be calculated as

$$\begin{aligned} P_2 &= \sum_{m=\frac{L_t-1}{2}}^{L_t-1} P_2(m)P(m) \\ &= \sum_{m=\frac{L_t-1}{2}}^{L_t-1} \frac{\binom{m}{\frac{L_t-1}{2}}}{2^m} \frac{(2L_t-1)!\left(\frac{L_t+1}{2}\right)!\left(\frac{L_t-1}{2}\right)!}{2^{L_t}(L_t-1)!L_t!}\bar{\gamma}^{-L_t} + o\left(\bar{\gamma}^{-L_t}\right) \\ &= \frac{(2L_t-1)!\left(\frac{L_t+1}{2}\right)!\left(\frac{L_t-1}{2}\right)!}{2^{L_t}(L_t-1)!L_t!}\bar{\gamma}^{-L_t} + o\left(\bar{\gamma}^{-L_t}\right), \end{aligned} \tag{38}$$

in which we utilize the identity

$$\sum_{m=\frac{L_t-1}{2}}^{L_t-1} \frac{\binom{m}{\frac{L_t-1}{2}}}{2^m} = 1. \tag{39}$$

The comparison between (38) and (33) reveals that, if L_t is odd, Scheme 1 suffers an asymptotic SNR loss equal to

$$\Delta_1 = \frac{10}{L_t}\log_{10}\frac{2^{L_t-1}\left(\frac{L_t+1}{2}\right)!\left(\frac{L_t-1}{2}\right)!}{L_t!} \quad \text{dB} \tag{40}$$

relative to the conventional $(L_t, 2; 1)$ TAS/STBC scheme.

B. Scheme 2

Similar to Scheme 1, we also have $n = L_t$ for Scheme 2. Since the second antenna is selected at random from the remaining $L_t - 1$ antennas, it is clear that

$$P(m) = \frac{1}{L_t-1}, \quad 1 \le m \le L_t - 1. \tag{41}$$

Therefore, we have

$$\begin{aligned} P_2 &= \sum_{m=1}^{L_t-1} P_2(m)P(m) \\ &= \sum_{m=1}^{L_t-1} \frac{1}{2^m}\frac{(2L_t-1)!}{2^{L_t}(L_t-1)!}\frac{1}{L_t-1}\bar{\gamma}^{-L_t} + o\left(\bar{\gamma}^{-L_t}\right) \\ &= \frac{\left[1-\left(\frac{1}{2}\right)^{L_t-1}\right](2L_t-1)!}{2^{L_t}(L_t-1)!(L_t-1)}\bar{\gamma}^{-L_t} + o\left(\bar{\gamma}^{-L_t}\right). \end{aligned} \tag{42}$$

Similarly, the asymptotic SNR loss of Scheme 2 relative to the conventional $(L_t, 2; 1)$ TAS/STBC scheme is quantified as

$$\Delta_2 = \frac{10}{L_t}\log_{10}\frac{2^{L_t-1}-1}{L_t-1} \quad \text{dB}. \tag{43}$$

C. Scheme 3

1) L_t is an Even Number: The instantaneous channel power gain associated with the i-th subset is given by

$$G_i = C_{2i-1} + C_{2i}, \quad 1 \le i \le \frac{L_t}{2}, \tag{44}$$

where C_i is defined in (1). For $L_r = 1$, it is clear that G_i are chi-squared variables with 4 degrees of freedom, whose pdf and cdf are given by [20, (2-1-110)]

$$p_{G_i}(x) = xe^{-x} \tag{45}$$

and [20, (2-1-114)]

$$P_{G_i}(x) = 1 - e^{-x}(1+x), \tag{46}$$

respectively. Following the subset selection criterion (9), the instantaneous channel power gain of the selected subset, denoted by $G_{(\frac{L_t}{2})}$, has the pdf written as

$$\begin{aligned} p_{(\frac{L_t}{2})}(x) &= \frac{L_t}{2}\left[P_{G_i}(x)\right]^{\frac{L_t}{2}-1} p_{G_i}(x) \\ &= \frac{L_t}{2}\left[1-e^{-x}(1+x)\right]^{\frac{L_t}{2}-1} xe^{-x}. \end{aligned} \tag{47}$$

Taking into account $\gamma = \frac{\bar{\gamma}}{2}G_{(\frac{L_t}{2})}$, we have the pdf of γ as

$$\begin{aligned} p_\gamma(\gamma) &= \frac{2}{\bar{\gamma}}p_{(\frac{L_t}{2})}\left(\frac{2\gamma}{\bar{\gamma}}\right) \\ &= \frac{L_t}{\bar{\gamma}}\left[1-e^{-\frac{2\gamma}{\bar{\gamma}}}\left(1+\frac{2\gamma}{\bar{\gamma}}\right)\right]^{\frac{L_t}{2}-1}\frac{2\gamma}{\bar{\gamma}}e^{-\frac{2\gamma}{\bar{\gamma}}} \\ &= \frac{2L_t\gamma}{\bar{\gamma}^2}\sum_{i=0}^{\frac{L_t}{2}-1}\sum_{j=0}^{i}(-1)^i\binom{\frac{L_t}{2}-1}{i}\binom{i}{j}\left(\frac{2\gamma}{\bar{\gamma}}\right)^j e^{-\frac{2(i+1)\gamma}{\bar{\gamma}}}. \end{aligned} \tag{48}$$

The MGF associated with γ is given by

$$\begin{aligned} M_\gamma(s) &= \int_0^\infty p_\gamma(\gamma)e^{s\gamma}d\gamma \\ &= \frac{2L_t}{\bar{\gamma}^2}\sum_{i=0}^{\frac{L_t}{2}-1}\sum_{j=0}^{i}(-1)^i\binom{\frac{L_t}{2}-1}{i}\binom{i}{j}\left(\frac{2\gamma}{\bar{\gamma}}\right)^j \\ &\qquad \times \int_0^\infty e^{\left[s-\frac{2(i+1)}{\bar{\gamma}}\right]\gamma}\gamma^{j+1}d\gamma \\ &= \frac{2L_t}{\bar{\gamma}^2}\sum_{i=0}^{\frac{L_t}{2}-1}\sum_{j=0}^{i}(-1)^i\binom{\frac{L_t}{2}-1}{i}\binom{i}{j}\left(\frac{2\gamma}{\bar{\gamma}}\right)^j \\ &\qquad \times (j+1)!\left[\frac{2(i+1)}{\bar{\gamma}}-s\right]^{-(j+2)}. \end{aligned} \tag{49}$$

Note that identity [22, (3.381-4)]

$$\int_0^\infty x^{\nu-1}e^{-\mu x}dx = \frac{1}{\mu^\nu}\Gamma(\nu), \quad \mathrm{Re}\{\mu\} > 0, \mathrm{Re}\{\nu\} > 0, \tag{50}$$

is utilized in the attainment of (49).

For BPSK, the BER expression can be written as [17]

$$\begin{aligned} P_2 &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_\gamma\left(-\frac{1}{\sin^2\theta}\right)d\theta \\ &= \frac{L_t}{2}\sum_{i=0}^{\frac{L_t}{2}-1}\sum_{j=0}^{i}(-1)^i\binom{\frac{L_t}{2}-1}{i}\binom{i}{j}(i+1)^{-(j+2)}(j+1)! \\ &\qquad \times \underbrace{\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{\sin^2\theta+\frac{\bar{\gamma}}{2(i+1)}}\right)^{j+2}d\theta}_{I_1}. \end{aligned} \tag{51}$$

Based on [17, (5A.4a)], the integral I_1 can be written as

$$\begin{aligned} I_1 = \frac{1}{2}\Bigg\{1 - \sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+2(i+1)}}\sum_{k=0}^{j+1}2^{-k}\binom{2k}{k} \\ \times\left[\frac{i+1}{\bar{\gamma}+2(i+1)}\right]^k\Bigg\}. \end{aligned} \tag{52}$$

Substituting (52) into (51) produces

$$P_2 = \frac{L_t}{4}\sum_{i=0}^{\frac{L_t}{2}-1}\sum_{j=0}^{i}(-1)^i\binom{\frac{L_t}{2}-1}{i}\binom{i}{j}(i+1)^{-(j+2)}(j+1)! \times\left\{1-\sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+2(i+1)}}\sum_{k=0}^{j+1}2^{-k}\binom{2k}{k}\left[\frac{i+1}{\bar{\gamma}+2(i+1)}\right]^k\right\}. \quad (53)$$

At high SNRs, (53) can be written as

$$P_2 = \frac{2^{\frac{L_t}{2}-1}(2L_t-1)!}{2^{2L_t-1}(L_t-1)!}\bar{\gamma}^{-L_t}+o\left(\bar{\gamma}^{-L_t}\right). \quad (54)$$

The asymptotic SNR loss of Scheme 3 for even L_t, relative to the conventional $(L_t, 2; 1)$ TAS/STBC scheme, can be quantified as

$$\Delta_3 = \frac{10}{L_t}\log_{10}2^{\frac{L_t}{2}-1} \quad \text{dB}. \quad (55)$$

2) L_t is an Odd Number: If L_t is odd, there are $\frac{L_t+1}{2}$ subsets in total. For the first $\frac{L_t-3}{2}$ subsets, the corresponding channel power gains

$$G_i = C_{2i-1}+C_{2i}, \quad 1\le i\le\frac{L_t-3}{2}, \quad (56)$$

are independent variables. We define $G_A = \max\{G_i\}$, $1 \le i \le \frac{L_t-3}{2}$.

For the last two subsets, the corresponding channel power gains are

$$G_{\frac{L_t-1}{2}} = C_{L_t-2}+C_{L_t-1} \quad (57)$$

and

$$G_{\frac{L_t+1}{2}} = C_{L_t-1}+C_{L_t}. \quad (58)$$

Note that $G_{\frac{L_t-1}{2}}$ and $G_{\frac{L_t+1}{2}}$ are not independent. We also define $G_B = \max\left\{G_{\frac{L_t-1}{2}}, G_{\frac{L_t+1}{2}}\right\}$. Following the selection criterion in (11), the instantaneous channel power gain of the selected subset, denoted by $G_{\left(\frac{L_t+1}{2}\right)}$, can be written as

$$G_{\left(\frac{L_t+1}{2}\right)} = \max\{G_A, G_B\}. \quad (59)$$

To obtain the cdf of $G_{\left(\frac{L_t+1}{2}\right)}$, we need to first derive the cdf's of G_A and G_B.

It is clear that the cdf of G_A is

$$P_{G_A}(x) = [P_{G_i}(x)]^{\frac{L_t-3}{2}} = \left[1-e^{-x}(1+x)\right]^{\frac{L_t-3}{2}}, \quad (60)$$

where $P_{G_i}(x)$ is defined in (46). Following Appendix II, the cdf of G_B can be written as

$$P_{G_B}(x) = 1-e^{-2x}-2xe^{-x}. \quad (61)$$

Therefore, it follows that the cdf and pdf of $G_{\left(\frac{L_t+1}{2}\right)}$ are given by

$$\begin{aligned} P_{\left(\frac{L_t+1}{2}\right)}(x) &= \Pr\{G_A<x, G_B<x\} \\ &= P_{G_A}(x)P_{G_B}(x) \\ &= \left[1-e^{-x}(1+x)\right]^{\frac{L_t-3}{2}}(1-e^{-2x}-2xe^{-x}), \end{aligned} \quad (62)$$

and

$$\begin{aligned} p_{\left(\frac{L_t+1}{2}\right)}(x) = \frac{L_t-3}{2}&\left[1-e^{-x}(1+x)\right]^{\frac{L_t-5}{2}} \\ &\times xe^{-x}\left(1-e^{-2x}-2xe^{-x}\right) \\ &+\left[1-e^{-x}(1+x)\right]^{\frac{L_t-3}{2}} \\ &\times\left(2e^{-2x}-2e^{-x}+2xe^{-x}\right), \end{aligned} \quad (63)$$

respectively.

Following the similar approach from (47) to (53), we have derived the exact BER expression for odd L_t. It can be written as

$$\begin{aligned} P_2 = &\frac{L_t-3}{2}\sum_{i=0}^{\frac{L_t-5}{2}}\sum_{j=0}^{i}(-1)^i\binom{\frac{L_t-5}{2}}{i}\binom{i}{j} \\ &\times\left[J_{\bar{\gamma}}(j+1,i+1)-J_{\bar{\gamma}}(j+1,i+3)-2J_{\bar{\gamma}}(j+2,i+2)\right] \\ &+2\sum_{i=0}^{\frac{L_t-3}{2}}\sum_{j=0}^{i}(-1)^i\binom{\frac{L_t-3}{2}}{i}\binom{i}{j} \\ &\times\left[J_{\bar{\gamma}}(j,i+2)-J_{\bar{\gamma}}(j,i+1)+J_{\bar{\gamma}}(j+1,i+1)\right], \end{aligned} \quad (64)$$

where

$$J_{\bar{\gamma}}(p,q) = \frac{p!}{2q^{p+1}}\left[1-\sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+2q}}\sum_{k=0}^{p}2^{-k}\binom{2k}{k}\left(\frac{q}{\bar{\gamma}+2q}\right)^k\right]. \quad (65)$$

At high SNRs, (64) can be written as

$$P_2 = \frac{(2L_t-1)!}{3\cdot 2^{\frac{3}{2}(L_t-1)}(L_t-1)!}\bar{\gamma}^{-L_t}+o\left(\bar{\gamma}^{-L_t}\right). \quad (66)$$

The comparsion between (66) and (33) reveals that, when L_t is odd, Scheme 3 suffers an asymptotic SNR loss equal to

$$\Delta_3 = \frac{10}{L_t}\log_{10}\frac{2^{\frac{1}{2}(L_t+1)}}{3} \quad \text{dB}, \quad (67)$$

relative to the conventional $(L_t, 2; 1)$ TAS/STBC scheme.

V. Trade-Off between Feedback Rate and Error Performance

Taking into account the trade-off between asymptotic error performance and feedback requirement, we will identify the relative merits of each scheme in this section, and endeavor to establish the practical application scenarios.

From (3) and (6), we find that

$$N - N_1 = 1, \quad (68)$$

for all L_t, except for $L_t = 6, 23$ and 91, [1] where $N = N_1$. In addition, the SNR loss of Scheme 1 in (34) and (40) has the property as following

$$\lim_{L_t\to\infty}\Delta_1 = 0 \quad \text{dB}. \quad (69)$$

By contrast, the SNR loss of Scheme 2 in (43) has the property of

$$\lim_{L_t\to\infty}\Delta_2 = 3 \quad \text{dB}. \quad (70)$$

[1]These are the only three solutions to $N = N_1$ for $L_t < 5000$.

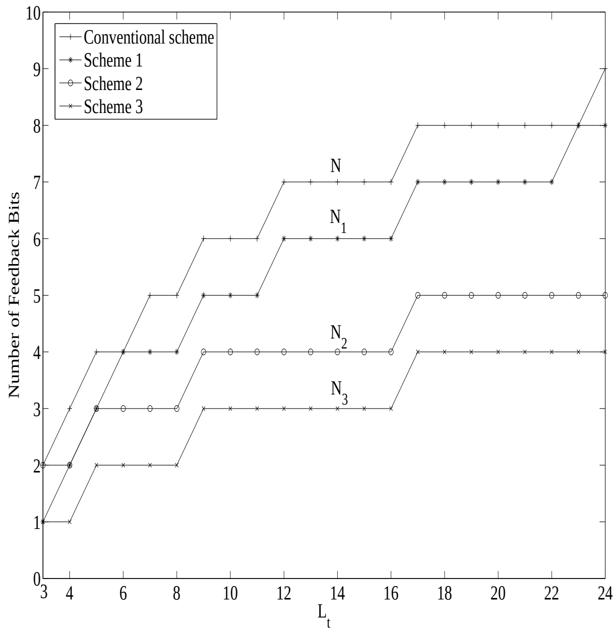

Fig. 3. The comparison of the number of feedback bits between the conventional and proposed antenna selection schemes.

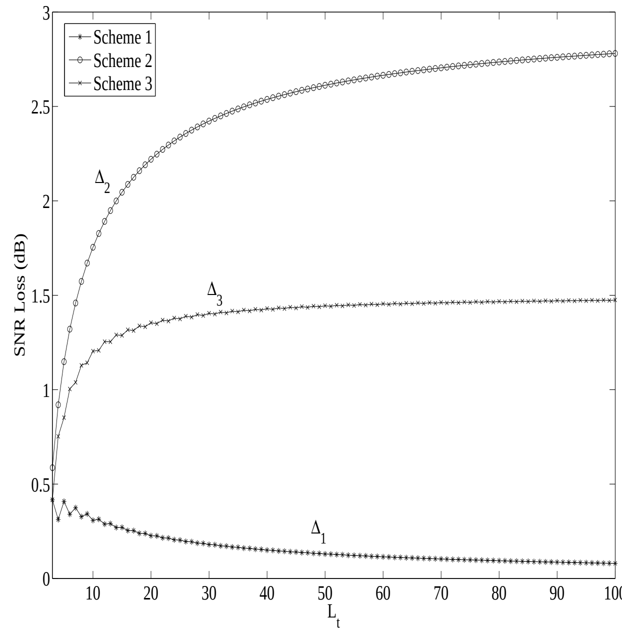

Fig. 4. Asymptotic SNR losses of the proposed schemes relative to the conventional scheme with BPSK Alamouti STBC, $3 \leq L_t \leq 100$, $L_r = 1$.

TABLE I

THE NUMBER OF FEEDBACK BITS AND ASYMPTOTIC SNR LOSSES FOR $3 \leq L_t \leq 8$

	L_t=3	L_t=4	L_t=5	L_t=6	L_t=7	L_t=8
N (bits)	2	3	4	4	5	5
N_1 (bits)	1	2	3	4	4	4
N_2 (bits)	2	2	3	3	3	3
N_3 (bits)	1	1	2	2	2	2
Δ_1 (dB)	0.4165	0.3123	0.4082	0.3402	0.3744	0.3276
Δ_2 (dB)	0.5870	0.9199	1.1481	1.3207	1.4588	1.5734
Δ_3 (dB)	0.4165	0.7526	0.8519	1.0034	1.0386	1.1289

We also have

$$\lim_{L_t \to \infty} \Delta_3 = 1.5 \quad \text{dB} \tag{71}$$

for the SNR loss of Scheme 3 in (55) and (67).

On the other hand, we have

$$\lim_{L_t \to \infty} \frac{N_2}{N} = \lim_{L_t \to \infty} \frac{N_3}{N} = \frac{1}{2}, \tag{72}$$

with the relationship $N_3 = N_2 - 1$.

Fig. 3 illustrates the number of feedback bits of the conventional and proposed antenna selection schemes. Fig. 4 presents the asymptotic SNR losses of the proposed schemes relative to the conventional scheme for $3 \leq L_t \leq 100$, assuming that BPSK Alamouti STBC is employed and $L_r = 1$. Since in most single link wireless systems, only a low to moderate number of transmit antennas are available, the numerical results for L_t of most practical interest are listed in Table I. We have several remarks as below.

- When L_t increases, the asymptotic SNR loss of Scheme 1 approaches zero, and it is upper bounded at $L_t = 3$ by 0.4165 dB. The reduction of feedback bits is always 1 bit, except for $L_t = 6$, 23 and 91, where there is no feedback reduction at all. For most values of L_t, Scheme 1 achieves a 1 bit feedback reduction at a negligible performance loss.
- Scheme 2 suffers asymptotic SNR losses lower bounded by 0.5870 dB when $L_t = 3$, and upper bounded by 3 dB when $L_t \to \infty$. Scheme 2 can reduce the number of feedback bits by almost half. However, at $L_t = 3$, there is no feedback rate reduction.
- Scheme 3 incurs asymptotic SNR losses lower bounded by 0.4165 dB at $L_t = 3$, and upper bounded by 1.5 dB when $L_t \to \infty$. It is important to point out that Scheme 3 needs one feedback bit less than Scheme 2, and suffers less SNR loss for any given L_t. Therefore, Scheme 3 is superior to Scheme 2, although Scheme 3 does not always include the best antenna in the selected subset while Scheme 2 does.
- From Table I, we can see that when $L_t = 3$, Schemes 1 and 3 have the same number of feedback bits and SNR loss. This is simply because for $L_t = 3$, Schemes 1 and 3 are exactly the same in essence. It is important to note that, as shown in Table I, Scheme 3 shows even better trade-off between error performance and feedback requirement for relatively small L_t. For example, at $L_t = 4$, Scheme 3 can reduce the number of feedback bits from three to one at a performance loss of 0.7526 dB when compared with the conventional scheme.

From the observations above, we conclude that Schemes 1 and 3 are more favorable in practical applications, and the appropriate application scenarios can be established as below.

- Scheme 1 is suitable for the applications where the

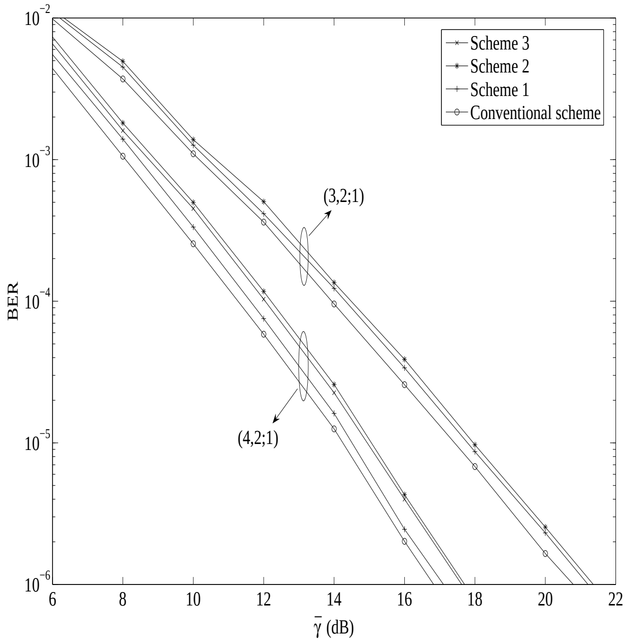

Fig. 5. Performance comparison between the conventional and proposed antenna selection schemes with BPSK Alamouti STBC, $L_r = 1$.

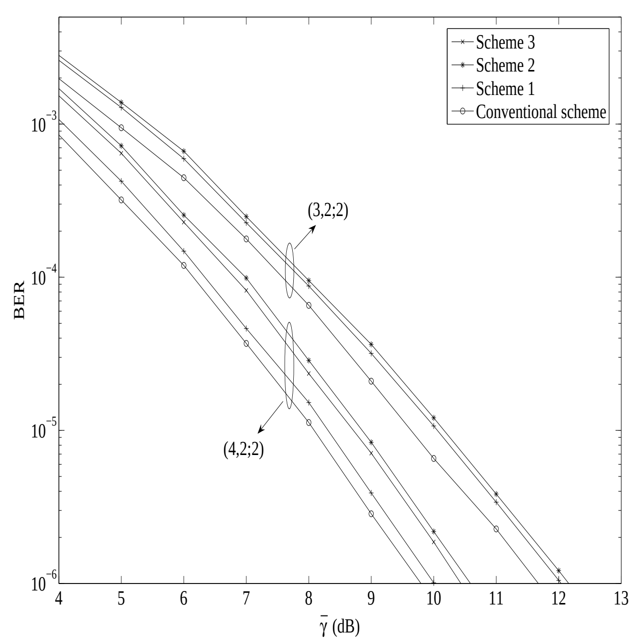

Fig. 6. Performance comparison between the conventional and proposed antenna selection schemes with BPSK Alamouti STBC, $L_r = 2$.

requirement for low feedback rate should not compromise the error performance significantly.

- For the wireless systems where feedback channel throughput is a major concern, Scheme 3 should be considered.

VI. Numerical Results

To substantiate the theoretic analysis in Section IV, some numerical results are provided. In the following discussion, SNR losses are measured at the BER of 10^{-6}.

Fig. 5 presents the simulated error performances of the conventional and proposed antenna selection schemes with BPSK Alamouti STBC in independent flat Rayleigh fading channels for a single receive antenna. It is shown that, compared with the conventional scheme, Schemes 1 and 2 suffer an SNR loss of 0.4 and 0.6 dB, respectively, for $L_t = 3$. When $L_t = 4$, Schemes 1, 2 and 3 suffer an SNR loss relative to the conventional scheme of 0.3, 0.9, and 0.8 dB, respectively. It is clear that the simulation results are consistent with analytical ones in Table I.

Although no analytical results are provided for two receive antennas, Fig. 6 depicts the similar comparison based on simulation. Performance losses similar to Fig. 5 are observed. Note that for $L_t = 3$, Scheme 1 and 3 are in essence the same. Therefore, the simulation results for Scheme 3 with $L_t = 3$ are not provided in Figs. 5 and 6.

VII. Conclusion

Three novel transmit antenna selection schemes with antenna selection restrictions were proposed to reduce the feedback rate. Analytical BER expressions for these three schemes with BPSK Alamouti STBC were derived based on the observation of the pdf of the instantaneous channel power gain, or by the MGF method. The asymptotic expressions at high SNRs show that the proposed schemes can achieve a full diversity order, as if all the available transmit antennas were used. The relative merits of these three schemes, in terms of trade-off between error performance and feedback rate, were discussed. And the appropriate application scenarios were identified. The results in this paper provide useful guidance for the design of transmit antenna selection systems with various feedback channel bandwidths and different requirements for quality of service.

Appendix I
Calculation of the Number of Feedback Bits

For a transmit antenna selection system, the feedback information can be the index of the selected subset, or the indexes of the selected antennas. In this Appendix, we will determine which method requires less number of feedback bits.

A. Conventional Scheme

For a conventional $(L_t, 2; L_r)$ transmit antenna selection system, if the index of the selected subset is fed back, the number of feedback bits required is given by

$$N_{0,1} = \lceil \log_2 \binom{L_t}{2} \rceil. \tag{73}$$

Alternatively, if the indexes of the two antennas within the selected subset are fed back, the number of feedback bits, denoted by $N_{0,2}$, is

$$N_{0,2} = 2\lceil \log_2 L_t \rceil \geq \lceil 2 \log_2 L_t \rceil = \lceil \log_2 L_t^2 \rceil. \tag{74}$$

Noting that

$$\binom{L_t}{2} = \frac{L_t(L_t-1)}{2} < \frac{L_t^2}{2}, \tag{75}$$

we find that

$$N_{0,1} < N_{0,2} \tag{76}$$

always holds. Therefore, we have

$$N = N_{0,1}. \tag{77}$$

This means that less feedback bits are required if the index of the selected antenna subset, instead of the indexes of the two selected antennas, is fed back to the transmitter. It easily follows that this conclusion can be extended to selected subset with more than two antennas.

B. Scheme 1

If the subset index is fed back to the transmitter, the number of feedback bits required is given by

$$N_{1,1} = \begin{cases} \lceil \log_2\left(\frac{L_t}{2}\cdot\frac{L_t}{2}\right)\rceil = \lceil 2\log_2 L_t - 2\rceil, & L_t \text{ even} \\ \lceil \log_2\left(\frac{L_t-1}{2}\cdot\frac{L_t+1}{2}\right)\rceil & \\ \quad = \lceil \log_2(L_t-1)+\log_2(L_t+1)-2\rceil, & L_t \text{ odd.} \end{cases} \tag{78}$$

By contrast, if the indexes of the two selected antennas are fed back, the number of bits is

$$N_{1,2} = \begin{cases} 2\lceil \log_2 \frac{L_t}{2}\rceil = 2\lceil \log_2 L_t - 1\rceil, & L_t \text{ even} \\ \lceil \log_2 \frac{L_t-1}{2}\rceil + \lceil \log_2 \frac{L_t+1}{2}\rceil & \\ = \lceil \log_2(L_t-1)-1\rceil + \lceil \log_2(L_t+1)-1\rceil, & L_t \text{ odd.} \end{cases} \tag{79}$$

Based on the fact that

$$\lceil x+y\rceil \le \lceil x\rceil + \lceil y\rceil, \tag{80}$$

we reach the conclusion that

$$N_{1,1} \le N_{1,2}. \tag{81}$$

Therefore we have

$$N_1 = N_{1,1}. \tag{82}$$

APPENDIX II
DERIVATION OF (61)

$$\begin{aligned} P_{G_B}(x) &= \Pr\{G_B < x\} \\ &= \Pr\left\{G_{\frac{L_t-1}{2}} < x, G_{\frac{L_t+1}{2}} < x\right\} \\ &= \Pr\{C_{L_t-2} < x - C_{L_t-1}, C_{L_t} < x - C_{L_t-1}\} \\ &= \int_0^x \Pr\{C_{L_t-2} < x-y\} \\ &\qquad \times \Pr\{C_{L_t} < x-y\}\, p_{C_i}(y)dy \\ &= \int_0^x [P_{C_i}(x-y)]^2 p_{C_i}(y)dy \\ &= \int_0^x \left[1-e^{-(x-y)}\right]^2 e^{-y}dy \\ &= 1 - e^{-2x} - 2xe^{-x}, \end{aligned} \tag{83}$$

where $p_{C_i}(y)$ and $P_{C_i}(y)$ are defined in (18) and (19), respectively.

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REFERENCES

[1] A. F. Molisch and M. Z. Win, "MIMO systems with antenna selection," *IEEE Microwave Mag.*, vol. 5, no. 1, pp. 46–56, Mar. 2004.
[2] D. A. Gore and A. J. Paulraj, "MIMO antenna subset selection with space-time coding," *IEEE Trans. Signal Processing*, vol. 50, no. 10, pp. 2580–2588, Oct. 2002.
[3] A. Ghrayeb and T. M. Duman, "Performance analysis of MIMO systems with antenna selection over quasi-static fading channels," *IEEE Trans. Veh. Technol.*, vol. 52, no. 2, pp. 281–288, Mar. 2003.
[4] A. Gorokhov, D. Gore, and A. Paulraj, "Receive antenna selection for MIMO flat-fading channels: theory and algorithms," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2687–2696, Oct. 2003.
[5] I. Bahceci, T. M. Duman, and Y. Altunbasak, "Antenna selection for multiple-antenna transmission systems: performance analysis and code construction," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2669–2681, Oct. 2003.
[6] X. N. Zeng and A. Ghrayeb, "Performance bounds for space-time block codes with receive antenna selection," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, pp. 2130–2137, Sep. 2004.
[7] A. Sanei, A. Ghrayeb, Y. Shayan, and T. M. Duman, "On the diversity order of space-time trellis codes with receive antenna selection over fast fading channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 7, pp. 1579–1585, July 2006.
[8] Q. Ma and C. Tepedelenlioglu, "Antenna selection for space-time coded systems with imperfect channel estimation," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 710–719, Feb. 2007.
[9] Z. Chen, J. Yuan, B. Vucetic, and Z. Zhou, "Performance of Alamouti scheme with transmit antenna selection," *Electron. Lett.*, vol. 39, no. 23, pp. 1666–1668, Nov. 2003.
[10] W. H. Wong and E. G. Larsson, "Orthogonal space-time block coding with antenna selection and power allocation," *Electron. Lett.*, vol. 39, no. 4, pp. 379–381, Feb. 2003.
[11] L. Yang and J. Qin, "Performance of Alamouti scheme with transmit antenna selection for M-ary signals," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3365–3369, Dec. 2006.
[12] M. Tao, Q. Li, and H. K. Garg, "Extended space-time block coding with transmit antenna selection over correlated fading channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3137–3141, Sep. 2007.
[13] Z. Chen, B. Vucetic, and J. Yuan, "Space-time trellis codes with transmit antenna selection," *Electron. Lett.*, vol. 39, no. 11, pp. 854–855, May 2003.
[14] J. Yuan, "Adaptive transmit antenna selection with pragmatic space-time trellis codes," *IEEE Trans. Wireless Commun.*, vol. 5, no. 7, pp. 1706–1715, July 2006.
[15] "Physical layer procedures (FDD)," 3rd Generation Partnership Project (3GPP), Tech. Rep. 25.214, 2005.
[16] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.
[17] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: John Wiley & Sons, 2004.
[18] M. Katz, E. Tiirola, and J. Ylitalo, "Combining space-time block coding with diversity antenna selection for improved downlink performance," in *Proc. IEEE VTC'01 Fall*, Atlantic City, NJ, Oct. 2001, pp. 178–182.
[19] Z. Wang and G. B. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1389–1398, Aug. 2003.
[20] J. G. Proakis, *Digital Communications*, 3rd ed. McGraw-Hill, 1995.
[21] H. A. David, *Order Statistics*. John Wiley & Sons, 1970.
[22] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. San Diego, CA: Academic, 2000.

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